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ARTICLE

Influential factors responsible for the effect of tax reduction on GDP

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Abstract The factors responsible for the effect of a tax reduction on GDP are analyzed using both agent-based modeling, based on the authors' previous study, and a derived set of equations for tax reduction multipliers, based on Morishima's economic linkage table. The findings are that, under the condition of balanced government finance, the tax reduction multiplier is determined by the difference between the increase in demand by consumers or firms as a result of the tax reduction and the decrease in demand by the government. To increase the effect of a tax reduction, it is necessary that the increased disposable income of consumers or firms, as a result of the tax reduction, is more likely to be used for consumption and investment. The analyzed factors that are proved to be indispensable to reproduce positive influence of a tax reduction in ABM are consistent with this mechanism.

Keywords Agent-based modeling · Tax reduction · Public expenditure

JEL Classification C63 · H210 · H220 · H230 · H250 · E620

1 Introduction

Agent-based modeling (ABM) is widely used in social simulations to explain or understand social phenomena (Terano 2008). One important area of research is the application of ABM to macroeconomic systems, although these systems are very

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complex and include various kinds of agents and interactions between them. When using ABM, it is considered important that the model be as simple as possible, based on the "KISS Principle", to understand the most essential mechanisms of the phenomena in question (Terano 2008). However, it is also important to consider all factors required to reproduce the desired phenomena, because the structure of the artificial system should be the same as that of the real system to enable the characteristics to emerge as they do in the real system. Establishing the factors essential to reproducing the desired characteristics of the system can be done using a series of computer experiments in which only one constituent factor of the model is changed at a time, while the other factors are held constant (Farmer and Foley 2009; Croson and Gächter 2010).

A number of ABM-based studies have focused on various macroeconomic concepts, such as business cycles, innovation, economic growth, the role of banks, monetary policies, industrial dynamics, and wealth inequality (Ashraf et al. 2011; Russo et al. 2007; Dosi et al. 2010; Bruun 2000). Most of these studies reported new findings, but structure of the model was different in each case. This makes it difficult to identify the crucial assumptions of each model and to what extent the assumptions are important in reproducing the phenomena being studied. Researchers have also developed relatively more practical models that simulate multiple-market economic structures as elaborately as possible (Raberto et al. 2011; Sprigg and Ehlen 2004). However, given the nature of these economic phenomena, these studies have not fully clarified the structural factors of the model that are important for reproducibility.

Another potential area for applying ABM in the real world is government policy formulation in areas such as tax reduction and public expenditure. According to Keynes' multiplier theory (Krugman and Wells 2009), government public expenditure and tax reductions are effective policies for promoting a macro-economy. However, the multipliers of these public policies are relatively small when compared with the values expected by the marginal propensity to consume in the Keynes' multiplier theory, the reason for which is not well understood. Authors recently clarified this reason, see Ogibayashi and Takashima (2017).

Motivated by this lack of understanding, the authors constructed a simple, artificial economic model consisting of consumers, three types of producers, a bank, and a government. The conditions required for the model to reproduce the positive influence of a tax reduction on GDP were then analyzed, with the intention of revealing and explaining the mechanism that makes the public policy multiplier so low. The findings showed that inefficiency in government expenditure, executive compensation, and internal funds for investment are all factors responsible for the positive influence of a tax reduction on GDP. Here, inefficiency in public expenditure is defined as the ratio of firm subsidies to the sum of firm subsidies and market purchases (Ogibayashi and Takashima 2013, 2014).

In the present study, additional simulations are conducted to clarify why the above factors are responsible for reproducing the positive influence of a tax reduction on GDP. In addition, we derive a set of equations for the tax reduction multiplier based on our revised version of Morishima's economic linkage table (Morishima 1984), and compare the influence of the above-mentioned factors with the results calculated using ABM.

In general, theoretical approaches in macroeconomics that assume complete equilibrium between demand and supply and that neglect the diversity of agents cannot describe complex systems well. However, these approaches have the advantage of being able to describe the mechanism behind the relationships among the influential factors. In explaining the influence of a tax reduction on GDP, the price in equilibrium would not be an important factor, whereas the flow of funds among agents would. In such cases, confirming whether the results calculated using ABM can be explained by theoretically derived equations could be an effective measure for validating ABM studies.

2 Simulation model

2.1 Outline of model

The ABM of the artificial economic system in the present study comprises consumers, producers, a bank, and a government as autonomous decision-making agents. Consumers and producers are each divided into three types of agent. Each agent is heterogeneous in its state variables and other parameters included in their action rules. Table 1 outlines the agents and their action rules.

2.2 Sequence of actions

The set of actions for each agent comprise period-based units, where one period is assumed to correspond to 1 month in real time. During each period, agents act and interact with each other according to a sequence of seven steps. At the end of the sequence for each period, a GDP value is calculated based on an input–output table obtained by summing each agent's account data. The details of the seven steps describing the sequence of agents' actions are as follows:

Table 1 Outline of agents and their action rules

Agent	Type	Output to be supplied	Product type to purchase
Consumer	Worker	The labor force for firms	Consumption goods
	Executive	Management for firms	
	Public worker	The labor force for government	
	Retailer	Consumption goods	Consumption goods, materials, equipment
Producer	Raw material maker	Material goods	Consumption goods, equipment
	Equipment maker	Equipment	–
Bank	–	The fund for producers	–
Government	–	Redistribution of wealth	Consumption goods

1. Agents pay any unpaid tax for the previous period. After paying tax, agents create a budget plan for consumption, paying wages, or public spending.
2. Raw material makers decide on the amount and price of products to be produced, produce several types of raw materials, and supply these to the material goods market.
3. Retailers decide on the amount and price of products to be produced, purchase raw materials in the material goods market, produce several types of consumption goods, and supply these products to the consumption goods market.
4. Consumers, retailers, raw material makers, and the government purchase products in the consumption goods market.
5. Retailers and raw material makers judge the necessity of investment based on total sales in previous periods and, if necessary, invest in equipment.
6. Each firm pays wages to employees and executive compensation to the executive, and the government pays wages to public workers.
7. Each agent settles their accounts using double-entry bookkeeping. They calculate their income and profit for the current term, and then determine the amount of tax to be paid based on these figures. If necessary, a retailer may dismiss a worker, depending on the profits in the current and previous periods, or may decide to stop producing a certain type of product altogether, depending on total sales.

2.3 Outline of agent's decision-making rules

2.3.1 Consumer agent behavior rules

Consumers create a budget for consumption E_b^t. This budget is defined as the sum of the terms based on after-tax income $I^t(1 - r_{i_tax})$ (represented by the Keynesian consumption function (Krugman and Wells 2009)), and a withdrawal ratio of r_{wd} times their bank deposit D_t, in each fiscal period t. The formula for the budget is shown in Eq. (1). Here, r_{i_tax} is the income tax rate, a is the consumer's basic consumption, and b is the marginal propensity to consume as per the Keynesian consumption function. The withdrawal ratio r_{wd} is selected randomly for each agent during each period.

$$E_b^t = a + bI^t(1 - r_{i\_\mathrm{tax}}) + r_{\mathrm{wd}}^t D^t \tag{1}$$

When purchasing products in the consumption market, consumers select and purchase products within the limit of their consumption budget according to the utility function for each class of products given by Eq. (2). Here, w_i is the weight of the utility for each class of product, i, which is randomly assigned to each agent, x_i is the number of products purchased, p_i is the price of a product, and α is an exponent ranging between 0 and 1. When there are goods of the same class of product available in the market at different prices, consumers select and purchase the cheapest available.

$$\max u = \sum_i w_i x_i^{\alpha} \quad \text{s.t.} \sum_i p_i^t x_i \leq E_b^t \tag{2}$$

2.3.2 Producer agent behavior rules

The retailers and raw material makers both decide on the amount and price of the products they will produce. The price p_i^t is determined as the product classification which each producer can produce according to the amount of product in stock and the amount bought in the market as shown in Eq. (3), where γ_{inc} is the ratio of price increase, γ_{dec} is the ratio of price decrease, s^{t-1} is the amount of goods in stock at the end of the previous period, se^{t-1} is the amount of goods sold at the end of previous period, and $p_i^{t-1}{}_{\text{ave}}$ is the average price of the goods bought in the market in the previous period.

$$p_i^t = \begin{cases} (1+\gamma_{\text{inc}})p_i^{t-1} & \text{if} \quad s_i^{t-1} = 0 \quad \text{and} \quad \text{se}_i^{t-1} > 0 \\ (1+\gamma_{\text{dec}})p_i^{t-1} & \text{if} \quad s_i^{t-1} > 0 \quad \text{and} \quad p_i^{t-1} > p_i^{t-1}{}_{\text{ave}} \end{cases} \tag{3}$$

The amount of production q_i^t is decided in such a way that the probability of being out of stock is less than 5% as shown in Eq. (4), where $q_{i_{\mu i}}$ is the average sales during the past ten periods, q_{i_σ} is the standard deviation of the total sales during the past ten periods, and ϵ is the ratio of the amount of production change. This is estimated based on the total sales over the most recent ten periods. If the estimated production is less than 70% of the production capacity, then the minimum amount of production is set at the 70% level.

$$q_i^t = \begin{cases} (q_{i_{\mu i}} + 1.65 q_{i_\sigma})(1+\epsilon) & \text{if} \quad s_i^{t-1} = 0 \quad \text{and} \quad \text{se}_i^{t-1} > 0 \\ (q_{i_{\mu i}} + 1.65 q_{i_\sigma})(1-\epsilon) & \text{if} \quad s_i^{t-1} > 0 \end{cases} \tag{4}$$

The production capacity Y is defined by the Cobb–Douglas production function (Krugman and Wells 2009) as shown in Eq. (5), where K is the number of units of equipment used in production, L is the number of employees, and α is assumed to be 0.25. In addition, A is a proportionality constant assigned randomly to each producer between a lower and upper limit. It is assumed that this value is peculiar to each producer, and represents that producer's technical capability.

$$Y_i(K, L) = A_i K^{\alpha} L^{1-\alpha} \tag{5}$$

Retailers and raw material makers initially have one unit of equipment and a specified number of employees. They can invest to increase their production capacity when production at maximum capacity continues beyond a critical period, based on the expected profit. They decide to invest when the expected financial benefit given by Eq. (6) is positive. Here, p_i is the price of a class of product i, c is the variable cost per unit of product, r_0 is the borrowing interest rate, F is the amount borrowed to buy one unit of equipment, N is the repayment period, and w is

the fixed wage per employee. It is assumed that the depreciation period of the equipment is the same as the repayment period.

$$\Delta\pi = \max_i \left[(p_i^t - c_i^t)\{Y_i(K+1,L) - Y_i(K,L)\} - (r_0 + 1/N)F\right] \tag{6}$$

When they decide to invest, half of the necessary funds are financed by the bank and the rest is financed internally by the firm. The funds for investment financed from the bank are repaid as a fixed amount in each period and for a constant number of consecutive repayment periods. During the repayment periods, additional investment is no longer allowed if the total number of investments exceeds a certain upper limit.

The equipment maker produces equipment in accordance with the requirements of the retailers and raw material makers within their production capacity limit. In the present study, the price of the equipment is assumed to be constant.

One executive and several workers are assigned to each of the producer agents. The producers pay a wage to the workers and the wage plus executive compensation to the executive in each period. The executive compensation in the real world comprises a paid salary, a bonus, and long-term incentives. In the present model, the paid salary is assumed to be the same as the wages paid to the workers, long-term incentives are ignored, and only the bonus is defined as executive compensation and is paid from after-tax profits. Wages comprise a fixed salary, randomly assigned to each employee between a lower and an upper limit, and a bonus given when the producer's profit is positive. The total spent on wages for each producer, E_w^t, is given in Eq. (7), where W_f is the total fixed salary, W_b^t is the total amount paid in bonuses, EC^t is the amount paid as executive compensation, π^t is the profit before tax, and AC is the accumulated earnings. In addition, W_b^t is defined as $r_b\pi^t$, where r_b is the bonus ratio, and EC^t is the executive compensation ratio multiplied by the after-tax profit, and is defined as $\pi^t(1-r_b)(1-r_{c_tax})r_e$, where r_{c_tax} is the corporate tax rate and r_e is the executive compensation ratio.

$$E_w^t = \begin{cases} W_f & \text{if } \pi^t < 0 \\ W_f + W_b^t & \text{if } \pi^t > 0 \text{ and } AC < 0 \\ W_f + W_b^t + EC^t & \text{if } \pi^t > 0 \text{ and } AC > 0 \end{cases} \tag{7}$$

2.3.3 Bank agent behavior rules

The bank retains the surplus money of the other agents in their respective bank accounts, earns interest on long-term and short-term loans, and pays wages to its employees and taxes to the government in line with its interest income. The bank lends money as long-term loans to producers in line with their demands for investment, charging a 3% interest rate. The bank also lends money as short-term loans to producers to meet their requirements when their working capital to pay fixed wages and/or purchase raw material becomes sufficiently small. In the present study, the initial funds available to the bank is set to be very large so that there is no

limitation on lending to producers, except in the case when long-term loans are not fulfilled during the repayment periods.

2.3.4 Government agent behavior rules

The government collects corporate tax and income tax, pays wages to public employees, and uses the resultant money for public expenditure according to their expenditure policy. Corporate tax is only collected when the profit of the producer is positive, and the tax rate is assumed to be constant. Income tax is also assumed to be constant and is collected according to the consumer's income. The wages of public employees are determined in each fiscal period so that they are equal to the average value of a private employee's combined fixed wage and bonus.

With regards to expenditure policies, market purchasing, firm subsidies, and combinations of the two are tested. Market purchasing is an extreme case of efficient public expenditure in which the government directly purchases goods at the market price. This policy corresponds to the case where the government places job orders with firms in a completely competitive situation at the market price. Firm subsidies are an extreme case of inefficient public expenditure in which the government distributes funds to producers evenly without any limitations on their use. This policy corresponds to the case in which the government places job orders at a much higher price than expected in the market or pays money for jobs that have no economic value. The efficiency in public expenditure is defined as the ratio of market purchasing to the total expenditure.

3 Simulation condition

A simulation program was constructed using C++ using an object-oriented method. The simulation conditions are given in Tables 2, 3 and 4. Table 2 shows the fixed parameters with values that remain constant during the simulation. Table 3 shows the initial conditions. These values are initially given by a uniform random number, but may change during each simulation run. In addition, the parameters displayed as

Table 2 Simulation condition at non-experimental level (parameter levels which does change during each run of simulation)

Maximum fiscal periods	360	Weight of utility	0.3–1.1
Number of consumers	150	Basic consumption	3000
Number of retailers	30	Marginal propensity to consume	0.7
Number of raw material makers	4	Fixed salary	7000–7500
Number of equipment makers	1	Ratio of increasing price	0.15
Number of banks	1	Ratio of decreasing price	0.1
Deposit interest rate	0.50%	Repayment period	120
Loan interest rate	3%	Investment value	500,000
Bonus ratio	75%	Critical flag number to quit production	20
Number of product classes	12	Critical flag number for dismissal	5

Table 3 Simulation condition at non-experimental level (initial conditions whose value may change during each run of simulation)

Consumer deposit	30,000–50,000
Capital of retailer and raw material maker	80,000–160,000
Capital of equipment maker	200,000–220,000
Capital of bank	96,000,000–104,000,000
Prices of raw material makers' products	130–160
Prices of retailers products	2850–3150
A in Eq. (5) for raw material maker	200–300
A in Eq. (5) for retailer	8–18

Table 4 Simulation conditions at experimental level

	Analysis of income tax rate			Analysis of corporate tax rate		
	Basic	High	Low	Basic	High	Low
Withdrawal ratio	0–0.2	0–0.5	0–0.8	0–0.5		
Executive compensation ratio	0–0.5			0.75	0.85	0.95
Income tax rate	10–30% (5% intervals)			20%		
Corporate tax rate	20%			10–30% (5% intervals)		
Efficiency in public expenditure	0–100% (10% intervals)			0–100% (10% intervals)		

a range of two values in Tables 2 and 3 are assigned a uniform random number within this range when the simulation starts, or during the simulation.

Table 4 shows the simulation parameters as experimental levels. These are constant, but change in each simulation run to clarify their influence on macroeconomic behavior in the artificial economic system.

The simulation conditions as experimental levels are divided into two categories: an analysis of the income tax rate and an analysis of the corporate tax rate. In the former category, the withdrawal ratio of deposit is changed during the experiment to change the marginal propensity to consume. In the latter category, the executive compensation ratio is changed to change the level of use of internal funds. In addition, in the latter case the number of investments during the overall simulation period is also analyzed as a function of the corporate tax rate.

4 Simulation results

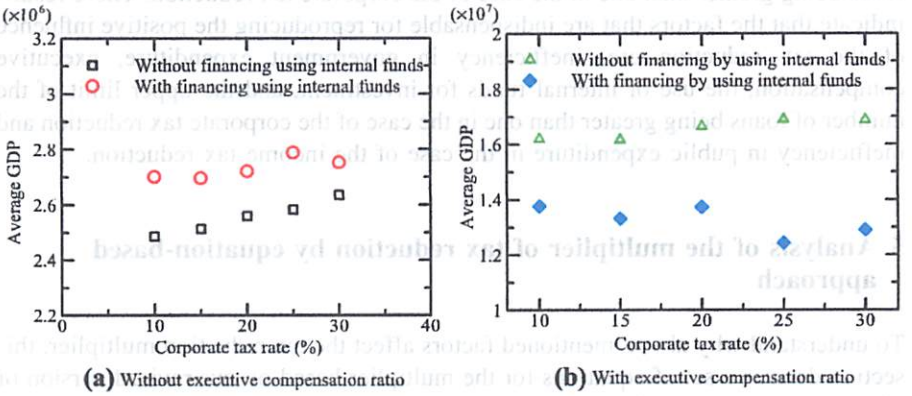
According to authors' previous studies (Ogibayashi and Takashima 2013, 2014), the inefficiency in public expenditure is an influential factor for reproducing the positive effect of both income and corporate tax reductions on GDP. However, in the case of the corporate tax reduction, other three factors are indispensable to reproduce the positive effect of tax reduction: executive compensation, financing using internal funds and the increased upper limit rule corresponding to the case where financing from the bank is not too restrictive (Ogibayashi and Takashima 2014).

The influence of executive compensation and financing using internal funds on the relationship between GDP and the corporate tax rate is shown in Fig. 1, where the upper limit of the number of loans is increased from one to three. It is noted that negative relationship between GDP and the corporate tax rate is only reproduced when executive compensation and financing using internal funds are employed in the model.

To understand the mechanism how these factors affect the relationship between GDP and the tax rate, we conducted additional simulations in the present study.

Figure 2 shows the dependency of the ratio of an increase in GDP as a result of the reduction in the tax rate on the market purchasing ratio and the consumers' withdrawal ratio. Here, the market purchasing ratio represents the efficiency in public expenditure, while the consumers' withdrawal ratio is an indicator of their marginal propensity to consume. The ratio of the GDP increase is defined as the ratio of an incremental increase in the original value when the income tax rate is reduced from 30 to 10%. In Fig. 2, each plot shows the interval between the first and third quartiles of the ratio of GDP increase, which is obtained by changing the initialization of the pseudo-random number generator in C++ in ten patterns. It is noted that, in Fig. 2, the ratio of GDP increase due to an income tax reduction becomes larger as the withdrawal ratio increases and the market purchasing ratio decreases.

Figure 3 shows the dependency of the ratio of a GDP increase as a result of a corporate tax reduction on the efficiency in public expenditure for various executive compensation ratios. As shown in Fig. 3, the ratio of the GDP increase as a result of a corporate tax reduction increases as the executive compensation ratio increases and as the efficiency in public expenditure decreases (that is, government spending becomes less efficient). To understand the reason why GDP is affected by the financing using internal funds, the number of investments is investigated as a function of the corporate tax rate for various levels of efficiencies in public expenditure, as shown in Fig. 4. In Fig. 4, tax reduction promotes investments. The reason for this tendency is that an increase in internal funds as a result of a tax reduction increases investments because the decision to invest depends on an amount of internal funds under the internal funds rule.



(a) Without executive compensation ratio **(b)** With executive compensation ratio

Fig. 1 Influence of executive compensation on the relationship between GDP and corporate tax rate

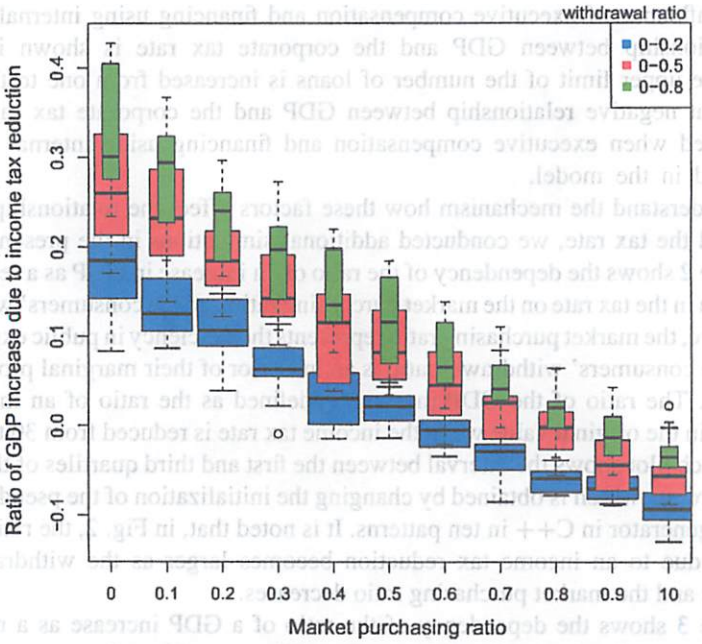


Fig. 2 Influence of the market purchasing ratio (efficiency in public expenditure) on the ratio of an increase in GDP caused by an income tax reduction, for various levels of withdrawal ratio which corresponds to the consumers' marginal propensity to consume

The results of ABM analysis are summarized as follows. The ratio of the GDP increase as a result of the reduction in the tax rate increases as the efficiency in public expenditure decreases in the case of both income tax and corporate tax reductions. This tendency is consistently obtained regardless of the seeds of random numbers. The ratio of the GDP increase also depends on the marginal propensity to consume in the case of the income tax reduction, and on the executive compensation ratio, the use of internal funds for investment, and the upper limit of the number of loans being greater than one in the case of the corporate tax reduction. These results indicate that the factors that are indispensable for reproducing the positive influence of the tax reduction are, inefficiency in government expenditure, executive compensation, the use of internal funds for investment, and the upper limit of the number of loans being greater than one in the case of the corporate tax reduction and inefficiency in public expenditure in the case of the income tax reduction.

5 Analysis of the multiplier of tax reduction by equation-based approach

To understand why above-mentioned factors affect the tax reduction multiplier, this section derives a set of equations for the multiplier based on our revised version of Morishima's economic linkage table (Morishima 1984).

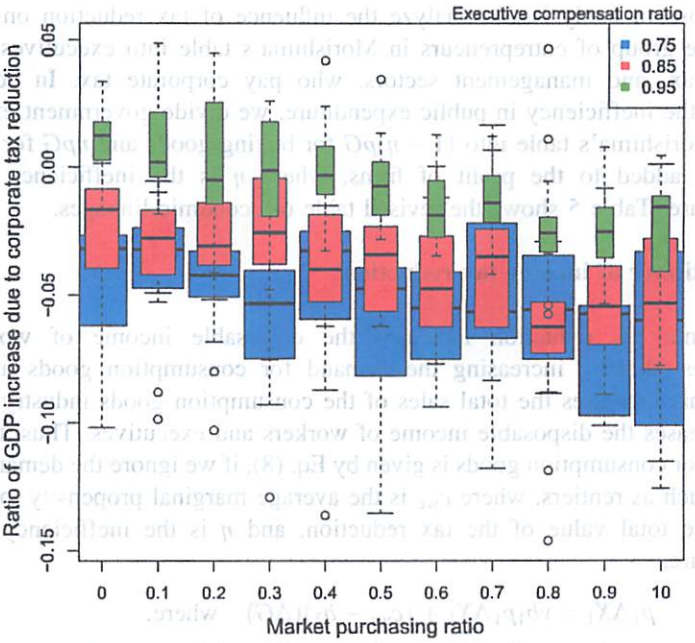


Fig. 3 Influence of the market purchasing ratio (efficiency in public expenditure) on the ratio of an increase in GDP caused by a corporate tax reduction, for various levels of executive compensation ratio which corresponds to the producers' marginal propensity to consume

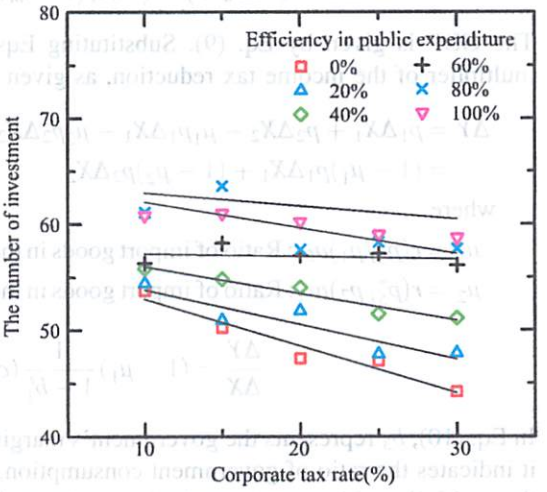


Fig. 4 Influence of corporate tax rate on the number of investment for different levels of the efficiency in public expenditure when executive compensation ratio is 0.95

In Morishima's table of economic linkages (Morishima 1984), individuals are divided into workers, entrepreneurs, rentiers, and firms and other sectors are divided into an investment sector, foreign trade, government, bank, and the rest. In addition, industrial outputs are assumed to be consumption goods and capital goods, the prices of which are assumed to be constant. Some corrections need to be made for

the purpose of analysis. To analyze the influence of tax reduction on GDP, we divide the group of entrepreneurs in Morishima's table into executives, who pay income tax, and management sectors, who pay corporate tax. In addition, to consider the inefficiency in public expenditure, we divide government expenditure pG, in Morishima's table into $(1-\eta)pG$ for buying goods and ηpG for subsidies, which is added to the profit of firms, where η is the inefficiency in public expenditure. Table 5 shows the revised table of economic linkages.

5.1 Multiplier of income tax reduction

An income tax reduction increases the disposable income of workers and executives, thereby increasing the demand for consumption goods as a direct result. This increases the total sales of the consumption goods industry, which in turn increases the disposable income of workers and executives. Thus, the overall demand for consumption goods is given by Eq. (8), if we ignore the demand of other sectors such as rentiers, where c_{we} is the average marginal propensity to consume, ΔG is the total value of the tax reduction, and η is the inefficiency in public expenditure.

$$\begin{aligned} p_1\Delta X_1 &= vb_1p_1\Delta X_1 + (c_{we} - b_3)(\Delta G) \quad \text{where,} \\ b_1 &= c_w(1-t_w)wa_{31}/p_1 + c_e(1-t_e)(1-t_m)\alpha mc_1/p_1 \\ b_3 &= c_e(1-t_e)(1-t_m)\alpha\eta + (1-\eta) \\ c_{\mathrm{we}} &= \{c_w(-\Delta t_w W) + c_e(-\Delta t_e(1-t_m)\alpha\Pi\}/\Delta G \\ \Delta G &= (-\Delta t_w W) + (-\Delta t_e(1-t_m)\alpha\Pi) \end{aligned} \tag{8}$$

The GDP is given by Eq. (9). Substituting Eqs. (8) into (9), we can obtain the multiplier of the income tax reduction, as given in Eq. (10).

$$\begin{aligned} \Delta Y &= p_1\Delta X_1 + p_2\Delta X_2 - \mu_1 p_1\Delta X_1 - \mu_2 p_2\Delta X_2 \\ &= (1-\mu_1)p_1\Delta X_1 + (1-\mu_2)p_2\Delta X_2 \\ &\text{where,} \\ \mu_1 &= r(p_5^*/p_1)a_{51}\text{: Ratio of import goods in the price of consumable goods} \\ \mu_2 &= r(p_5^*/p_2)a_{52}\text{: Ratio of import goods in the price of durable goods.} \end{aligned} \tag{9}$$

$$\frac{\Delta Y}{\Delta X} = (1-\mu_1)\frac{1}{1-b_1'}(c_{\mathrm{we}} - b_3). \tag{10}$$

In Eq. (10), b_3 represents the government's marginal propensity to consume because it indicates the ratio of government consumption. Therefore, Eq. (10) indicates that the multiplier of income tax reduction is determined by the difference between the average marginal propensity to consume of consumers and that of the government.

Assuming that $c_w = c_e$ and substituting into Eq. (10) typical values from our simulation presented in the previous section, such as $t_w = t_e = 0.2$, $wa_{31}/p_1 = wa_{32}/p_2 = 0.6$, $m = 0.25$, $c_1/p_1 = c_2/p_2 = 0.8$, $t_1 = t_2 = 0$, the multiplier of the income tax reduction is represented as a function of the efficiency in

Table 5 Revised table of economic linkages

		(1) Consumption goods industry	(2) Capital goods industry	(3) Workers, w	(4) Executives, e	(5) Rentiers, r	(6) Management sector, m
1	Consumption goods	$-p_1X_1$		$p_1D_1^w$	$p_1D_1^e$	$p_1D_1^r$	
2	Capital goods		$-p_2X_2$				
3	Wages	$wa_{31}X_1$	$wa_{32}X_2$	$-W$			
4	Depreciation	$p_4a_{41}X_1$	$p_4a_{42}X_2$				
5	Imports	$rp_5^*a_{51}X_1$	$rp_5^*a_{52}X_2$				
6	Taxes	$t_1(1+m)c_1X_1$	$t_2(1+m)c_2X_2$	t_wW	$t_e\alpha(1-t_m)\Pi$	$t_r(\overline{A}^r + r\overline{B}_*^r)$	$t_m\Pi$
7	Profits	mc_1X_1	mc_2X_2		$-\alpha(1-t_m)\Pi$		$\alpha(1-t_m)\Pi + \gamma - \Pi)$
8	Interest					$-\overline{A}^r$	
9	Bond					$p_b\delta B^r$	
10	Time deposit					δQ^r	
11	Foreign exchange					$r(p_h^*\delta B_*^r - B_*^r$	
12	Money			$L^w - \overline{M}^w$	$L^e - \overline{M}^e$	$L^r - \overline{M}^r$	$L^m - \overline{M}^m$

		(7) Investment sector, i	(8) Foreign trade, f	(9) Government, g	(10) Bank, b	(11) Central bank, c	(12) Foreign exchange stabilization fund, s
1	Consumption goods		p_1E_1	$(1-\eta)G_1$			
2	Capital goods	p_2I_2	p_2E_2	$(1-\eta)G_2$			
3	Wages			wN^g	wN^b		
4	Depreciation	$-H$					
5	Imports		$-rp_5^*F$				
6	Taxes			$-T$			
7	Profits	$-\gamma$		$\eta(G_1+G_2)$			

Table 5 continued

		(7) Investment sector, i	(8) Foreign trade, f	(9) Government, g	(10) Bank, b	(11) Central bank, c	(12) Foreign exchange stabilization fund, s
8	Interest	$-\overline{B}^i$	$-\overline{B}^f$	$-\overline{B}^g$	$-\overline{A}^b$	$-\overline{A}^c$	
9	Bond	$p_b\delta B^i$	$p_b\delta B^f$	$p_b\delta B^g$	$p_b\delta B^b$	$p_b\delta B^b$	
10	Time deposit				δQ^b	δQ^b	
11	Foreign exchange		$r(D^f_\$ - S^f_\$)$				$rD^s_\$$
12	Money	$L^i - \overline{M}^i$			$L^b - \overline{M}^b$	$-\delta M^c$	$-\delta M^s$

public expenditure and of the marginal propensity to consume, as shown in Fig. 5. It is noted that this tendency coincides with the relationship obtained from the simulation and shown in Fig. 2.

5.2 Multiplier of corporate tax reduction

The multiplier of corporate tax reduction would be very small if internal funds that increase as a result of the tax reduction are assumed not to be used for investment. We derive the equation for the multiplier based on Table 5, assuming that β times the amount of the tax reduction is used for investment. This investment increases the demand for capital goods, which increases the total sales of firms. This increases the disposable income of workers and executives, and therefore, increases the demand for consumption goods. The government decreases public expenditure to compensate for the decrease in tax revenue under the balanced budget condition. The overall demand for consumption goods and capital goods are represented by Eqs. (11) and (12), respectively. Substituting Eqs. (11) and (12) into (10), we obtain Eq. (13), which represents multiplier of the corporate tax reduction.

$$p_1\Delta X_1 = b_1 p_1 \Delta X_1 + b_2' p_2 \Delta X_2 + (b_4 - b_3)\Delta G$$

where,

$$\begin{aligned} b_1 &= c_w(1-t_w)a_{31}w/p_1 + c_e(1-t_e)(1-t_m)\alpha mc_1/p_1 \\ b_2' &= c_w(1-t_w)a_{32}w/p_1 + c_e(1-t_e)(1-t_m)\alpha mc_2/p_2 \\ b_4 &= c_e(1-t_e)\alpha, \quad \alpha\text{: Ratio of executive compensation.} \end{aligned} \tag{11}$$

$$p_2\Delta X_2 = p_2 I_2 = \beta\Delta G$$

where, β: Ratio of consumption for investment to the amount of tax reduction. (12)

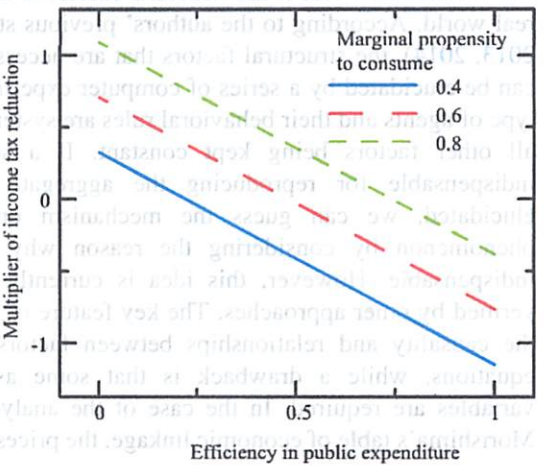


Fig. 5 Influence of efficiency in public expenditure and marginal propensity to consume on the multiplier of income tax reduction

$$\begin{aligned}\frac{\Delta Y}{\Delta G} &= (1-\mu_1)(b_2'\beta + b_4 - b_3)(1-b_1) + (1-\mu_2)\beta \\ &= A\beta + (b_4 - b_3)(1-\mu_1)/(1-b_1) \\ &\text{where,} \quad A: \text{ Multiplier of firm's investment.}\end{aligned} \tag{13}$$

In Eq. (13), the terms β and b_4 represent the ratio of consumption for investment and executives' marginal propensity to consume, respectively, and their sum represents the ratio of consumption to the total amount of the tax reduction. Therefore, Eq. (13) indicates that the multiplier of corporate tax reduction is determined by the difference between the substantial marginal propensity to consume of firms and that of the government. Assuming that $c_w = c_e = 0.9$, $t_w = t_e = 0.2$, $wa_{31}/p_1 = wa_{32}/p_2 = 0.6$, $m = 0.25$, $c_1/p_1 = c_2/p_2 = 0.8$, $t_1 = t_2 = 0$, and substituting these values into Eq. (11), the multiplier of corporate tax reduction is represented in Fig. 6. It is noted that the multiplier increases as the ratio of executive compensation increases, and as the efficiency in public expenditure decreases. This tendency agrees with Fig. 3. It is also noted that the multiplier increases as the value of β increases. This tendency agrees with the relationship shown in Fig. 4. The reason why the use of internal funds for investment is an indispensable condition for reproducing a positive influence of tax reduction on GDP in our simulation (Ogibayashi and Takashima 2014) is that firms must use part of their increased internal funds for investment as a result of the corporate tax reduction.

6 Discussion

To understand the influential factors and the mechanism underlying the positive influence of a reduction in the tax rate on GDP, we adopted two approaches, agent-based analysis, and equation-based analysis.

The key feature of the agent-based analysis is that an aggregate phenomenon can emerge in the artificial society as a result of the agents' actions and interactions, as it does in the real world if the structural factors in the model are similar to those in the real world. According to the authors' previous studies (Ogibayashi and Takashima 2013, 2014), the structural factors that are necessary to reproduce the phenomenon can be elucidated by a series of computer experiments in which factors such as the type of agents and their behavioral rules are systematically changed one by one with all other factors being kept constant. If a set of structural factors that are indispensable for reproducing the aggregate phenomenon under concern is elucidated, we can guess the mechanism underlying the emergence of the phenomenon by considering the reason why this set of structural factors is indispensable. However, this idea is currently our conjecture and needs to be verified by other approaches. The key feature of the equation-based analysis is that the causality and relationships between factors are easily understood from the equations, while a drawback is that some assumptions relating to aggregate variables are required. In the case of the analysis in the present study based on Morishima's table of economic linkage, the prices of consumption goods and capital

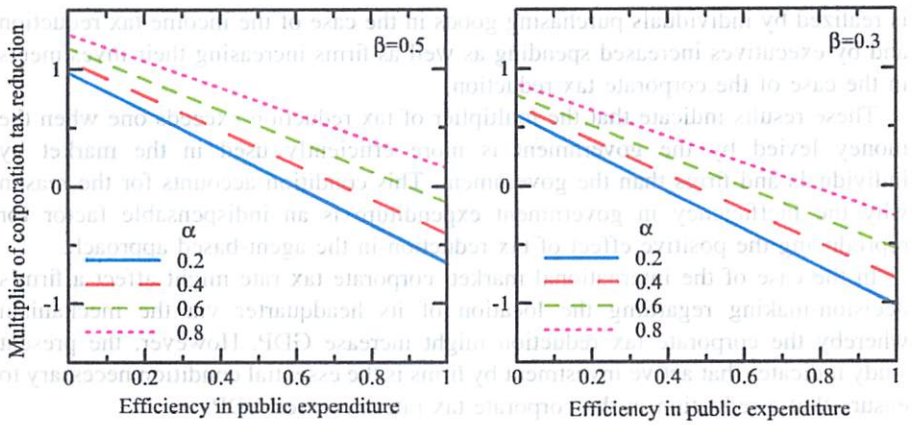


Fig. 6 The influence of the efficiency in public expenditure, α and β, on the multiplier of corporate tax reduction. α executive compensation ratio, β consumption ratio for investment

goods are assumed to be constant, and the funds in the market are assumed to be in equilibrium.

Although both approaches have their advantages and drawbacks, it is noted that the influences of structural factors on the effect of tax reduction are qualitatively coincident in both approaches. That is, the positive effect of tax reduction on GDP increases with a decrease in the efficiency in public expenditure under both approaches (see Figs. 2 and 5 regarding the income tax reduction and Figs. 3 and 6 regarding the corporate tax reduction). The marginal propensity to consume shown in Fig. 6 corresponds to the withdrawal ratio shown in Fig. 2, and both approaches consistently show that the positive effect of the income tax reduction on GDP increases with an increase in the marginal propensity to consume. The parameter α in Fig. 6 corresponds to the executive compensation ratio, and both approaches consistently show that the positive effect of a reduction in the corporate tax rate on GDP increases with an increase in the executive compensation ratio. The parameter β in Fig. 6 is the consumption ratio for investment and corresponds to the financing rule using internal funds for investment in Fig. 1. As indicated in Fig. 6, the multiplier of the corporate tax reduction increases with increasing β, implying that the positive influence of the corporate tax reduction becomes greater if the greater amount of surplus money increased as a result of tax reduction is used for investment. This tendency is consistent with that shown Fig. 1, where the positive influence of a reduction in the tax rate is only reproduced in the model when internal funds are assumed to be used for investment. The upper limit of the number of loans which is greater than one, is another indispensable factor for reproducing the positive effect of a reduction in the corporate tax rate because it corresponds to the degree of credit rationing, noting that a necessary condition for increasing β is that bank financing is not overly restrictive

Based on these results, it is considered that a reduction in the tax rate results in an increase in GDP when the surplus money of consumers' or firms' increased as a result of the tax reduction is effectively consumed in the market. This consumption

is realized by individuals purchasing goods in the case of the income tax reduction and by executives increased spending as well as firms increasing their investments in the case of the corporate tax reduction.

These results indicate that the multiplier of tax reduction exceeds one when the money levied by the government is more efficiently used in the market by individuals and firms than the government. This condition accounts for the reason why the inefficiency in government expenditure is an indispensable factor for reproducing the positive effect of tax reduction in the agent-based approach.

In the case of the international market, corporate tax rate might affect a firm's decision-making regarding the location of its headquarter via the mechanism whereby the corporate tax reduction might increase GDP. However, the present study indicates that active investment by firms is the essential condition necessary to ensure that a reduction in the corporate tax rate increases GDP.

7 Conclusion

The factors responsible for the effect of a tax reduction on GDP were analyzed using both ABM, based on the authors' previous study, and using a theoretical derivation of a set of equations for tax reduction multipliers, based on our revised version of Morishima's economic linkage table. We obtained the following findings.

1. In the ABM analysis, the ratio of the GDP increase due to tax reduction increases as the efficiency in public expenditure decreases in the case of both income tax and corporate tax reductions. The ratio of the GDP increase also depends on the marginal propensity to consume in the case of income tax reduction, and on the executive compensation ratio and the frequency of investments in the case of corporate tax reduction.
2. The equations for the tax reduction multiplier, which are derived in the present study, show that the multipliers have the same dependency on influencing factors observed in the ABM analysis. It is found that the most important factor in the case of the corporate tax reduction is the ratio of the increased consumption for investment to the total amount of the tax reduction.
3. The derived equations suggest that, under the balanced finance condition, the tax reduction multiplier is determined by the difference between the increased consumption of goods and investment by consumers or firms, and the decreased demand by the government. The critical amount of consumption for a positive multiplier becomes larger if public expenditure becomes more efficient.

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