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Influence of Inefficiency in Government Expenditure on the Multiplier of Public Investment

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Abstract The multiplier of public investment has been expected to far exceed 1, owing to the indirect influence of public spending. However, it has been reported that actual multipliers for a real economy are sometimes <1; the reason for this has not been adequately explained in the literature. This study analyzes the influence of inefficient public expenditure on gross domestic product, using both an agent-based model and a theoretical derivation of the equation for the multiplier of public investment, the latter of which is based on our revised version of Morishima's economic linkage table. The use of both of these instruments indicates that gross domestic product decreases with an increase in the inefficiency of public expenditure, which is defined as the ratio of firm subsidies to the government's total expenditure. The multiplier of public investment becomes <1 when the degree of inefficiency is sufficiently large, and the ratio of the firm's investment spending to the total amount of subsidy funding is sufficiently small. A multiplier lower than 1 is thought to appear when the degree of inefficiency in public expenditure is sufficiently large and firms are reluctant to invest; much of the surplus amount of subsidized funds can be deposited into a bank account, thus reducing the money stock in the market.

Keywords Agent-based modeling · Inefficiency of government expenditure · Multiplier · Public investment

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1 Introduction

There has been widespread interest in the economic influences of fiscal policy, especially since the great recession of 2007–2009 (Whalen and Reichling 2015). In promoting an economy while reducing government deficits, much attention has been paid to understanding how fiscal policy affects the economy; this attention has been reflected in the ongoing debate over the size of the fiscal multiplier (Whalen and Reichling 2015).

According to traditional economic theory, the multiplier of public investment has been defined as the inverse of the marginal propensity to save (Krugman and Wells 2009). It is also well known, however, that the actual multiplier of public investment is much smaller than that which theory predicts (Baum et al. 2012; Bruckner and Tuladhar 2010; Murata et al. 2005; OECD 2009; Perroti 2004; Sugimoto 2008). The multiplier of an actual system is generally estimated through the use of three methods: the use of macroeconometric forecasting models, time-series models, and dynamic stochastic general equilibrium models (Whalen and Reichling 2015). Although estimated values vary greatly, it is widely recognized that the multiplier can be <1, or even negative in some cases (Baum et al. 2012; Bruckner and Tuladhar 2010; OECD 2009; Perroti 2004; Sugimoto 2008). For example, Sugimoto (2008) estimated the multiplier of public investment in the Japanese economy and reported that the average multiplier for the 1955–2011 period ranged from 0.76 to 1.38. Perroti (2004) estimated multipliers of public investment for five advanced countries and reported that the estimated multipliers ranged between -0.88 and 5.46, thus indicating that the multiplier can be not only <1 but also negative. This discrepancy has been partially explained in the literature as stemming from the influences of other factors, such as taxation and transfer payments to households (Krugman and Wells 2009).

In addition, Morishima (1984) derived an equation with which to obtain the multiplier of public investment, based on reasonable assumptions under which the multiplier can be as low as 1.45, thus indicating that the multiplier is also affected by the ratio of imports and firms' ways of distributing earnings surpluses, among other things. However, these factors cannot on their own explain why the multiplier can be <1. Bruckner et al. (Bruckner and Tuladhar 2010) studied the multiplier of the Japanese economy and discussed the influences of such factors that contribute to the low multipliers as the inefficient allocation of public investment, overinvestment and high capital stock that leads to low marginal productivity, and the influence of crowding out. However, the most responsible factor among them has not been specified because of a lack of evidence. Although there have been many arguments on this matter (Baum et al. 2012; Bruckner and Tuladhar 2010; OECD 2009; Perroti 2004; Whalen and Reichling 2015), there is still no conclusive consensus as to why the multiplier might be <1. In this sense, there has been a dearth of research that quantitatively explains why actual multipliers may sometimes be <1, or even negative.

One interesting study is that of Perroti (2004), who pointed out that public investment might be particularly prone to political pressure and loaded with so-called pork-barrel projects that have no economic rationale. In more extreme cases, this can foster downright corruption and rent-seeking activities (Perroti 2004). Although this explanation seems reasonable, his idea is not supported by the results of any quantitative analysis.

Meanwhile, agent-based modeling (ABM) is a sound approach for studying why the multiplier of public investment might be <1, because it is a bottom-up approach in the sense that macro phenomena emerge in an artificial system modeled on a computer as a result of interactions between agents in a way similar to that of an actual system, and therefore, ABM can deal with heterogeneity, individual agents' bounded rationality, and non-equilibrium dynamics in social systems. One criticism of ABM relates to the validation of ABM and points out that the macro phenomenon that emerges in ABM is insensitive to parameter values and it would therefore be difficult to derive the necessary conditions for the model to exhibit specific macro behaviors as pointed out in the literature (Marks 2007).

According to the authors' previous study (Takashma and Ogibayashi 2014), however, input conditions in ABM can be divided into parameter values and the system structure of the model and the macro phenomenon is quite sensitive to the system structure of the model, which is characterized by such factors as the types of agents, the behavioral rules of agents, and the fields where agents develop their activities. In addition, there is one-to-one correspondence between the macro phenomenon, its underlined mechanism, and the model structure, the latter of which is indispensable in reproducing the phenomenon. By studying this model structure that is indispensable in reproducing the desired phenomena, in terms of a series of computer experiments, we can elucidate the underlying mechanism of the occurrence of the macro phenomenon (Takashma and Ogibayashi 2014)

In relation to the model structure that is required to reproduce the influence of public policies, we have developed an agent-based model of a macroeconomic system and analyzed the model structure that is indispensable in reproducing the positive influence of a corporate tax reduction and that includes inefficiency in public expenditure, executive compensation, the use of internal funds for investment, and not-too-severe credit creation. The strongest factor among them is inefficiency in government expenditure, where said inefficiency is defined as the ratio of a firm subsidy to the total value of government expenditure (Ogibayashi and Takashima 2013, 2014). Here, the inefficiency in government expenditure in our previous and present studies corresponds to the degree to which the government pays an excessive amount of money to firms compared with its economic value. This payment of an excessive amount of money is a kind of transfer payment or firm subsidy that is made in the real economy in various ways behind the mask of economic policy packages such as industrial promotion and the protection of domestic industry.

With respect to previous research on the ABM approach on the fiscal multiplier, there is a very recent paper that analyzed the influence of the credit market on the fiscal multiplier (Napoletano et al. 2015). However, that research focused on the influence of government spending on the impact of and recovery from the exogenously given bankruptcy shock of a household in a specifically designed artificial society that does not seem to be an imitation of the real economic society in the sense that, in the real system, every macro phenomenon emerges as a result of interactions between decision-making agents. Moreover, it does not analyze the influence of the way of government spending.

The present study introduces the idea of inefficiency in government expenditure and conducts a series of computer experiments, based on the ABM of artificial economic systems developed by the authors (Ogibayashi and Takashima 2009, 2013, 2014), to analyze the influences of inefficient public expenditure on the multiplier of public investment and the gross domestic product (GDP). The system structure of the model in the present study is essentially the same as that of the model that reproduced the influence of a corporate tax cut. In addition to the ABM approach, mathematical equations for the multiplier of public investment are derived according to our revision of Morishima's economic linkage table; also discussed are the mechanism for the influence of inefficiency in government expenditure on the multiplier of public investment, and the reason why the multiplier is sometimes <1.

2 Simulation Model

2.1 Outline of Model

In the present study, the ABM of an artificial economic system involves consumers, producers, a bank, and a government; all are autonomous decision-making agents. It is assumed that consumers and producers can each be divided into three types of agent. Each agent is heterogeneous in terms of its state variables and other parameters included in its action rules.

Table 1 outlines the agents included in the model. The agents' behavioral rules are outlined as follows.

Consumer — Consumers work and obtain wages at one of the producers, the bank, or the government, then they pay income tax and purchase consumption goods within the limits of the budget for consumption. They select and purchase products in the market so as to maximize their utility. When there are products of the same class available in the market at different prices, they select the cheapest among them.

Producer — Producers hire consumers, produce and sell products, and pay wages and corporate tax. The retailers and raw-material makers decide both

Table 1 Outline of agents and their behavioral rules

Agent	Type	Output to be supplied	Product type to be purchased
Consumer	Worker	Labor force for firms	Consumption goods
	Executive	Management for firms	
	Public workers	Labor force for government	
Producer	Retailer	Consumption goods	Materials, equipment
	Raw-material maker	Material goods	Equipment
	Equipment maker	Equipment	–
Bank	Bank	Fund for producers	–
Government	Government	Redistribution of wealth	Consumption goods

the amount and price of each class of product, taking into account the amount of stocks in each period. Each of them invests in equipment to increase its production capacity, when sales are good for more than a critical period. The equipment maker produces equipment in line with the requirements of the retailers and raw-material makers.

Bank — The bank keeps the surplus money of other agents in their respective bank accounts, and lends money.

Government — The government collects tax from other agents, pays wages to public workers, and spends the remaining money as public expenditure in line with expenditure policy.

Each of the agents has state variables which are renewed in each time step as a result of interaction with other agents. Variables that are endogenously calculated in the present model are divided into two categories, namely aggregate variables such as the GDP and state variables of each agent such as the balance of deposits, both of which correspond to variables of real systems.

2.2 Sequence of Actions

Each agent's set of actions has period-based units, where one period is assumed to correspond to 1 month in the real system. During each period, agents act and interact with each other, in a seven-step sequence. In addition, each agent records transaction data according to the double-entry bookkeeping method. At the end of the sequence for each period, each agent settles the account and a GDP value is calculated according to an input–output table obtained by summing the account data of each agent. Details of the agents' actions in each of the seven sequential steps are described below.

1. Agents pay accrued tax for the previous period. After paying tax, agents make a budget plan for consumption or public spending, including wages to be paid to workers.
2. Raw-material makers decide the amount and price of products to be produced, generate products from several types of raw materials, and supply them to the material goods market.
3. Retailers decide the amount and price of products to be produced, purchase raw materials from the material goods market, generate products from several types of consumption goods, and supply those products to the consumption goods market. Here, the price and amount of production are affected by the results of purchasing raw materials from the material goods market prior to production.
4. Each of the consumers, retailers, raw-material makers, and the government purchases products from the consumption goods market, in line with its consumption preferences.
5. Each firm pays wages to employees and executive compensation to its executives; the government pays wages to public workers.
6. Retailers and raw-material makers assess the necessity to invest, on the basis of total sales in the previous periods; if necessary, they will invest in equipment by purchasing a set of equipment from an equipment maker.

7. Each agent settles its accounts employing a double-entry bookkeeping method, where the income or profits for the current term is calculated; from this, the amount of tax to be paid is determined. If necessary, each retailer dismisses a worker on the basis of profits in the current and previous periods, or decides to stop production of a certain type of product on the basis of its total sales.

2.3 Outline of Agents' Decision-Making Rules

2.3.1 Consumer Agent Behavioral Rules

Consumers make a budget for consumption E_b^t that is defined as the sum of a part of income I^t (as defined by the Keynesian consumption function) and the withdrawal ratio r_{wd} multiplied by the bank deposit D^t in each fiscal period t (Eq. (1)). r_{i_tax} is the income tax rate, a is basic consumption, and b is the marginal propensity to consume as per the Keynesian consumption function. The withdrawal ratio r_{wd} is given at random for each agent.

$$E_b^t = a + bI^t(1 - r_{i\_tax}) + r_{wd}^t D^t \tag{1}$$

When purchasing products in the consumption goods market, consumers select and purchase products within the limits of the budget for consumption. The class and number of products to be purchased are decided so as to maximize utility, according to the utility function given by Eq. (2). Here, w_i is the weight of utility for each type of class i, which is randomly assigned for each agent; x_i is the number of products to purchase; p_i is the price of the product; and α is an exponent of x_i that ranges from 0 to 1. When there are goods of the same class available in the market at different prices, consumers will select and purchase the cheapest among them.

$$\max u = \sum_i w_i x_i^\alpha \quad \text{s.t.} \sum_i p_i^t x_i \le E_b^t \tag{2}$$

Each of the consumers makes two types of decisions in each period, and decides not only the class of products to purchase but also the amount to purchase.

With respect to deciding the class of products, each consumer type has his or her own utility for each class of products; this too is a function of the number of products to be bought. He or she decides to purchase the combination of the class and number of consumption goods so as to maximize the utility function.

2.3.2 Producer Agent Behavioral Rules

The retailers and raw-material makers both decide on the amount and price of the product they will produce according to the production capacity and amount of stock at the end of the previous period. The price of a product in a product class is lowered, raised, or not changed depending on how much of the product is in stock at the end of the previous period. Production levels are decided in such a way that the probability of being out of stock is <5%. This is estimated according to the total sales over the most recent 10 periods. The production capacity, Y, is defined by the Cobb–Douglas production function as a function of the number of units of equipment and the number of employees. The power exponent for the number of units of equipment is assumed to be 0.25. Refer to the authors' previous study for details (Ogibayashi and Takashima 2014).

Each retailer and raw-material maker initially has one unit of equipment and a specified number of employees. Each will invest to increase its production capacity when production at maximum capacity continues beyond a critical number of periods, according to the expected profit. It decides to invest when the expected financial benefit π_K given by Eq. (3) is positive. Here, p_i is the price of a product of class i, c is the variable cost per unit product, r_0 is the borrowing interest rate, F is the amount borrowed to buy one unit of equipment, N is the repayment period, and w is the fixed wage per employee. It is assumed that the depreciation period of the equipment is the same as the repayment period.

$$\Delta\pi_K = \max_i[(p_i^t - c_i^t)\{Y_i(K+1, L) - Y_i(K, L)\} - \{(1+r_0)/N\}F] \tag{3}$$

When each decides to invest, half the necessary funds are financed by the bank; the rest is financed by the firm's internal funds. The investment funds financed from the bank are repaid as a fixed amount in each period and for a constant number of consecutive repayment periods. During the repayment periods, additional investment is no longer allowed when the total number of loans exceeds a certain upper limit.

The equipment maker produces equipment in accordance with the requirements of the retailers and raw-material makers and within its production capacity limit. In the present study, the price of the equipment is assumed to be constant.

One executive and several workers are initially assigned to each of the producers. In each period, each producer pays a wage to each of the executive and workers. The wage is composed of a fixed salary that is constant in each period and a bonus. In addition to the wage, an executive compensation is paid to the executive from the after-tax profit. The bonus and executive compensation are determined according to the profit in the previous period. The fixed salary is assigned randomly to each of them by generating a uniform random number between a lower limit and upper limit Refer to the authors' previous study for details (Ogibayashi and Takashima 2014).

2.3.3 Bank Agent Behavioral Rules

The bank lends money in the form of long-term loans to producers (in line with their demands for investment) while charging a 3% interest rate. The bank also lends money to producers in the form of short-term loans so that they may meet their requirements when their working capital to pay fixed wages and/or purchase raw materials becomes sufficiently depleted. In the present study, the bank is initially given a very large quantity of funds such that there is no limitation on lending to producers, except in cases where long-term loan payments are not made during the repayment period.

2.3.4 Government Agent Behavioral Rules

The government collects corporate tax and income tax, pays wages to public employees, and uses the remaining money to fund public expenditures in line with expenditure policy. The wages of public employees are determined in each fiscal period such that they are equal to the average income of private employees.

Market purchasing, firm subsidies, and combinations of these are tested as expenditure policies. Market purchasing is an extreme case of efficient public expenditure, in which the government directly purchases goods in the market at the market price. This policy corresponds to the public expenditure, where the government places job orders with firms in a completely competitive situation and at the market price. A firm subsidy is an extreme case of inefficient public expenditure in which the government evenly distributes funds to producers without placing any restrictions on their use; much of the funds thus distributed could be transferred to the bank account without being used in the market. This policy corresponds to any kind of transfer payment to firms that is economically unreasonable, examples of which are public expenditures where the government places job orders at a price level much higher than that expected in the market, or pays money for jobs that have no economic value.

To estimate the multiplier of public investment, the government spends an additional expenditure ΔG during the specified periods in the simulation; this expenditure is assumed to be financed by the issuance of government bonds. For simplification, however, additional expenditure for public investment ΔG is simply added to the total budget obtained from the tax revenue, without taking into account the bond market or relevant transactions.

3 Simulation Conditions

For the analysis of the influence of inefficiency in government expenditure on the multiplier of public investment, a series of computer experiments is systematically conducted, changing the specific conditions one by one while other conditions are held constant. A simulation program was constructed using C++, the program code of which is presented on GitHub (https://github.com/ogilabnet/ABM-Macroeconomics/). The simulation conditions at the experimental levels are shown in Table 2. As given in Table 2, five factors are employed as experimental levels; these are ① the upper limit on the number of loans, which serves as an index for the credit restriction, ② the threshold on the decision making on investment, which is the critical number of periods during which firms take into account the total sales for the decision making on investment (i.e., if the production at maximum capacity with the state being sold out continues beyond these critical periods, it is assumed that the firm will decide to invest), ③ the timing of public investment in business cycles, ④ the total amount of funds used in public investment, and ⑤ the degree of inefficiency in government expenditure.

These experimental factors and levels are described in greater detail as follows.

The upper limit on the number of loans is the maximum number of loans that the firms can have within the same period; this is assumed to be two or three, with two corresponding to severer credit restrictions.

Table 2 Simulation conditions at the experimental levels

Factor number	Experimental factor	Experimental level
①	Upper limit on the number of loans	2, 3
②	Threshold for the decision making on investment	10, 20
③	Timing of public investment	Before bottom, after bottom, growing, peak, declining
④	Amount of funds for investment	Small, middle, large
⑤	Inefficiency in government expenditure	0–100%

The threshold for the decision making on investment is the number of periods with no stocks. If the condition of the state being out of stock continues under maximum capacity production, then the firm will decide to invest. This number corresponds to a firm's willingness to invest, which is assumed to be 10 or 20, with 10 corresponding to firms being positive toward investment.

The timing of public investment within business cycles is the period during which the government starts investment; this is assumed to be just before the bottom of a cyclic variation in GDP (i.e., the business cycle), just after the bottom, during the middle of the growing stage, during the middle of the declining stage, or near the peak. Here, the total amount of funds for investment is divided into 12 parts, and each increment of public investment is continuously added to the budget over the 12 periods to purchase capital goods or subsidize the firms, depending on the degree of inefficiency in government expenditure. Here, the exact period of the peak or bottom of the business cycle varies across different experimental levels of the upper limit on the number of loans (two or three; two levels) and the threshold on the decision making on investment (10 or 20; two levels). Therefore, the exact period for the start of investment is set differently, in line with the four experimental levels; this was analyzed in a prior experiment on the condition without public investment.

The total amount of funds for public investment is also changed to be small, mid-range, or large, in line with the experimental levels; it is set to 1.25, 1.5, or 2.0 times, respectively, the average tax revenue over 360 periods for each of the experimental levels. The standard value of the average tax revenue was previously calculated for each of the four aforementioned experimental levels, under the condition of no public investment and the assumption that the degree of inefficiency in government expenditure is zero.

The inefficiency in government expenditure is changed from 0 to 100%, at 10% intervals for each of the aforementioned 60 conditions (i.e., two levels for each of ① and ②, five levels for ③, and three levels for ④).

The multiplier due to public investment is calculated as per Eq. (4), in which the increment of the increase in GDP due to public investment is assumed to be the average of the difference between the pre-and post-investment GDPs.

Table 3 Simulation conditions at the non-experimental level

(a) Parameter values of the base run			
Maximum number of fiscal periods	360	Weight of utility	0.3–01.1
Number of consumers	150	Basic consumption	3000
Number of retailers	30	Marginal propensity to consume	0.7
Number of raw-material makers	6	Fixed salary	7000–7500
Number of equipment makers	1	Ratio of increasing price	0.15
Number of banks	1	Ratio of decreasing price	0.1
Withdrawal ratio	0–0.5 at random	Repayment period	120
Loan interest	3%	Investment value	500000
Bonus ratio	75%	Critical flag number at which to quit production	b20
Executive compensation rate	95%	Lower limit of production	70% of its capacity
Number of product classes	12		
(b) Initial conditions whose value may change during each run of simulation			
Consumer deposit	30,000–50,000		
Capital of retailer and raw material maker	80,000–160,000		
Capital of equipment maker	200,000–220,000		
Capital of bank	96,000,000–104,000,000		
Prices of raw-material makers' products	130–160		
Prices of retailers' products	2850–3150		

$$\Delta GDP = \frac{\sum_{t=start}^{start+12} (GDP^t - GDP^{start})}{12\Delta G} \tag{4}$$

The simulation conditions with respect to the base condition other than the experimental factors are given in Table 3.

4 Simulation Results

Figure 1 shows the variation in GDP during the entire simulation period; it shows how the business cycle is driven by the repetition of bank financing for firm investment and subsequent repayment (Ogibayashi and Takashima 2009). The GDP in each period, as well as the average GDP during the entire simulation period, decreases as the inefficiency in government expenditure increases, as shown in Figs. 1 and 2. This

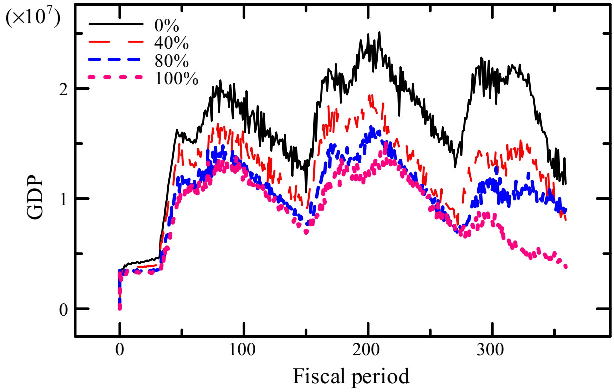

Fig. 1 Influence of inefficiency in government expenditure, which is changed from 0 to 100%, on the variation in GDP; the limit on the number of loans is two and the investment threshold is 20

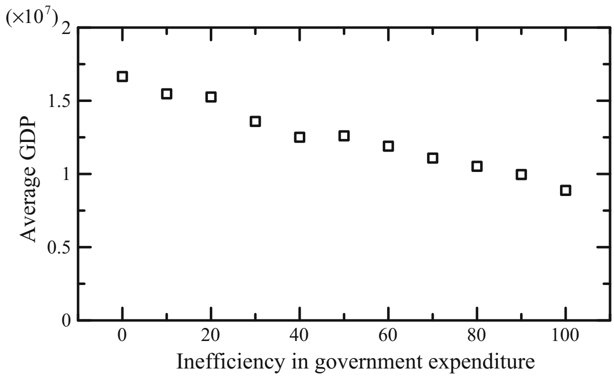

Fig. 2 Influence of inefficiency in government expenditure on the GDP average over 360 periods; the limit on the number of loans is two and the investment threshold is 20

tendency is caused by the fact that part of the money transferred to the firms in the form of firm subsidies as a result of inefficient government expenditures will be saved in the firms' bank accounts and not consumed in the market, thus resulting in a decrease in the money stock in the market. Direct evidence of the tendency that deposits increase with an increase in the inefficiency in government expenditures is discussed later and presented in Fig. 4.

Inefficiency in government expenditure also affects the multiplier of public investment. An example of this tendency is shown Fig. 3, in which public investment starts just before the bottom of a business cycle, which is near the 140th period in Fig. 1. It is noted that in Fig. 3 the multiplier decreases with an increase in the degree of inefficiency in government expenditure; the multiplier becomes <1 when the inefficiency exceeds 50%, when the amount of funds available for public investment is large. When that amount is small, the multiplier becomes negative when the inefficiency is sufficiently large; however, this is only because the amount of funds for investment is not sufficiently large, compared with the amplitude of the cyclic variation in GDP.

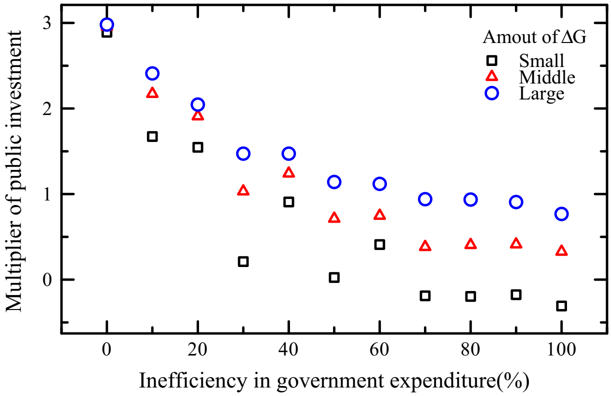

Fig. 3 Multiplier of public investment as a function of the inefficiency in government expenditure, calculated under the condition that the public investment is conducted before the bottom stage; the limit on the number of loans is two and the investment threshold is 20

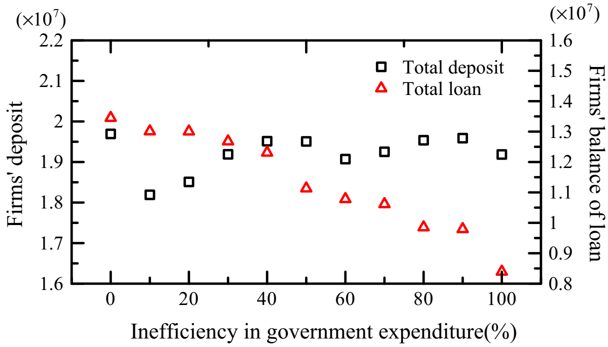

Fig. 4 Influence of the inefficiency in government expenditure on the amount of firms' deposits and loans, calculated under the condition that the public investment, the amount of which is large, is made before the bottom stage; the limit on the number of loans is two and the investment threshold is 20

Hereafter, therefore, only the calculated result in the case of a large amount of funds for investment will be presented.

To understand the reason why the multiplier decreases with an increase in the inefficiency in government expenditure, the total amounts of firms' deposits and firms' loan balances were analyzed. As shown in Fig. 4, with an increase in the inefficiency in government expenditure, the loan balance decreases and the firms' bank deposits increase. This result suggests that at least a part of the funds transferred to the firms as a result of inefficient government expenditure will be used by firms to increase their bank deposits, as well as to reduce the balance of their loans through repayment; both of these results will lead to a decrease in the money stock in the market, and thus reduce the multiplier. The multiplier decreases with an increase in the inefficiency in public expenditure, because this tendency strengthens as the inefficiency in government expenditure increases.

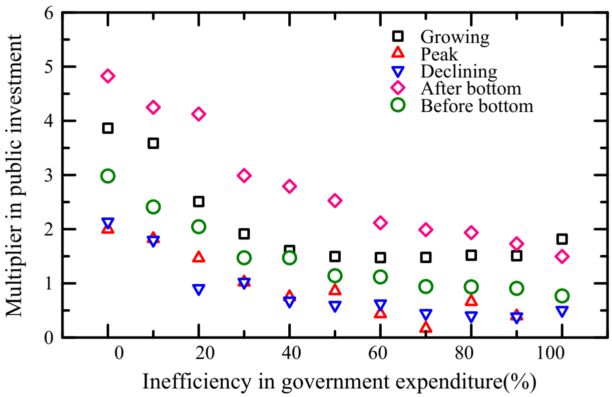

Fig. 5 The multiplier as a function of the inefficiency in government expenditure for various timings of government expenditure in the business cycle, calculated under the condition that the amount of public investment is large; the limit on the number of loans is two and the investment threshold is 20

The tendency for the multiplier to decrease as the degree of inefficiency in government expenditure increases is consistently observed when the timing of public investment changes among the five experimental levels (Fig. 5). Regarding the different timings of public investment, the multiplier in the case of investment during the growing stage or during the stage just after the bottom is larger than multipliers under the other conditions. To explain this tendency, average numbers of investment by firms during the periods of public investment in the simulation were examined. It was found that average numbers during the growing stage and just after the bottom are 31.3 and 23.8, respectively, which are larger than those of other cases (i.e., those during the peak stage, declining stage, and just before the bottom are 18.3, 0.09, and 11.6 respectively). Therefore, one of the reasons for the tendency is that investment more frequently occurs in such timing cases than in other cases, thus helping public investment increase GDP more effectively.

The willingness of firms to invest also affects the multiplier. The index for the willingness of investment in the present simulation is introduced in terms of the threshold on investment decision making (Table 2), where a lower value of the threshold corresponds to firms being more positive toward investment. As shown in Fig. 6, the multiplier increases as the willingness of firms to invest increases.

5 Analysis of the Multiplier of Public Investment Based on an Economic Linkage Table

Our previous study (Ogibayashi and Takashima 2013) revealed that the positive multiplier of tax reduction can be reproduced in ABM only when some degree of inefficiency in government expenditure is assumed, and the multiplier decreases with an increase in the inefficiency in government expenditure. The reason for this dependency of the multiplier on the inefficiency in government expenditure is that the multiplier of tax reduction becomes positive only when the government's marginal propensity to con-

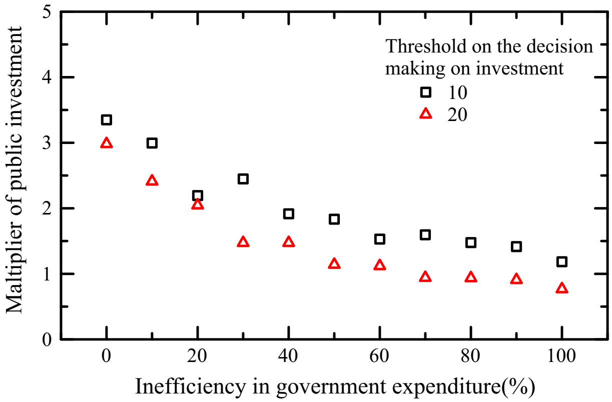

Fig. 6 Influence of the threshold on investment decision making on the relationship between the multiplier of public investment and inefficiency in government expenditure, calculated under the condition that the amount of public investment is large and the public investment is conducted before the bottom stage

sume is lower than that of the private sector. The latter decreases with an increase in the inefficiency of government expenditure, because part of the subsidized funds is likely to be deposited in the firms' bank accounts and not be consumed in the market (Ogibayashi and Takashima 2014). In addition, the previous study on the influence of a corporate-tax reduction on GDP revealed that to reproduce a positive multiplier in corporate-tax reduction, the model structure needs to include not only the inefficiency in government expenditure but also executive compensation and the use of internal funds in investment.

Based on the findings of the previous studies, the present study analyzes the influence of inefficiency in government expenditure taking the ABM approach. The study elucidates, as seen in the previous section, that the multiplier of public investment is greatly affected by inefficiency in government expenditure. It is also found that the multiplier can be <1 when the inefficiency in government expenditure is sufficiently large.

Meanwhile, the use of a mathematical model in which the economic equilibrium is assumed is another approach with which to analyze the multiplier of public investment; it has an advantage in terms of obtaining insights into underlying mechanisms. Morishima (1984) derived a set of equations with which to determine the multiplier of public investment using an economic linkage table, assuming an industrial society where no primary sector of industry exists and all raw materials are imported. Morishima's model, however, requires revisions in some respects. In Morishima's table of economic linkages, there is no management sector within a firm that pays corporate tax, and the entrepreneur thus has the dual role of being both an executive and the management sector. In addition, Morishima's model does not take into account inefficiency in government expenditure; as such, the government expenditure for public investment is assumed to be directly consumed in the market, at the market price.

The present study therefore divides the entrepreneurs in Morishima's study into executives who receive executive compensation and pay income tax, and those who comprise a management sector that pays corporate tax. In addition, to consider the inefficiency of public expenditure, we divide government expenditure pG in Morishima's table into $(1 - \eta)pG$ for buying goods and ηpG for a subsidy, the latter of which is added to the profit of firms; here η is inefficiency in government expenditure. Table 4 is the revised table of economic linkages.

Using Table 4, let us derive a set of equations for the multiplier of public investment, on the basis of Morishima's assumptions and procedure for deriving the equations. According to Morishima, we can make the following assumptions. Columns 1 and 2 in Table 4 represent price-determining equations in the production sectors; the sum of each column is zero. Other columns show the budget equations for each economic agent, and the sum of each column can be set to zero if the circulation of funds is assumed to be in equilibrium. In addition, the sum of each row can be set to zero, if we assume that ① the prices of consumption goods and capital goods are each constant, because production is flexibly adjusted and there is no excess demand or supply in either industry, ② the monetary market is in equilibrium, and ③ the stock market is not taken into account, the supply and demand in both the bond market and the foreign exchange market are in equilibrium, the exchange rate is constant, and other factors are assumed negligible.

The present study also assumes that neither rentiers nor the bond market in Table 4 exist.

Under these assumptions, an aggregate increase in the production of capital goods as a result of investment in the public or private sector may increase the income and consumption of each economic agent, and thus induce an aggregate increase in the production of consumption goods. This results in an increase in GDP, which is calculated as the sum of the increased demand in both consumption and capital goods, minus the total payment for imported materials.

In Table 4, X_1 and X_2 represent the supplied amounts of consumption goods and capital goods, D_1^w, D_1^e, D_1^r, D_1^m, E_1, and G_1 the demands for consumption goods by workers, executives, rentiers, the management sector, foreign trade, and government, p_1 and p_2 the prices of consumption goods and capital goods inclusive of sales tax, w the wage rate, W the obtained wage of workers, p_4 the price of capital service, r the exchange rate, p_5^* the price of imported raw materials in foreign currency (e.g., dollars), m the mark-up rate, t_1 and t_2 the sales taxes for consumption goods and capital goods, t_w, t_e, and t_r the income tax rates of workers, executives, and rentiers, t_m the corporate tax rate, Π a firm's profit, $\alpha\Pi$ the executives' income, I_2 the demand for capital goods due to a firm's investment, and $\gamma_i \Pi$ a firm's budget for investment. Other symbols are the same as those used in Morishima's economic linkage table (see Morishima 1984).

5.1 Derivation of the Equation for the Multiplier of Public Investment

5.1.1 Multiplier of Public Investment for Capital Goods Where All Required Funds are Financed by Government Debt

Let us derive a set of equations for the multiplier of public investment, while we assume that public investment is conducted in the capital goods market, take into

Table 4 Economic linkage table used in the present analysis, which is a revised version of Morishima's (1984) table

		(1) Consumption goods industry	(2) Capital goods industry	(3) Workers w	(4) Executives e	(5) Rentiers r	(6) Management sector m
1	Consumption goods	$-p_1 X_1$		$p_1 D_1^w$	$p_1 D_1^e$	$p_1 D_1^r$	$p_1 D_1^m$
2	Capital goods		$-p_2 X_2$				
3	Wages	$wa_{31} X_1$	$wa_{32} X_2$	$-W$			
4	Depreciation	$p_4 a_{41} X_1$	$p_4 a_{42} X_2$				
5	Imports	$rp_5^* a_{51} X_1$	$rp_5^* a_{52} X_2$				
6	Taxes	$t_1(1+m)c_1 X_1$	$t_2(1+m)c_2 X_2$	$t_w W$	$t_e \alpha \Pi$	$t_r(\overline{A}^r + r\overline{B}_*^r)$	$t_m \Pi$
7	Profits	$mc_1 X_1$	$mc_2 X_2$		$\alpha \Pi$		$(\alpha(1 - t_m) + \gamma_i - 1)\Pi$
8	Interest					$-\overline{A}^r$	
9	Bond					$p_b \delta B^r$	
10	Time deposit					δQ^r	
11	Foreign exchange					$r(p_b^* \delta B_*^r - B_*^r$	
12	Money			$L^w - \overline{M}^w$	$L^e - \overline{M}^e$	$L^r - \overline{M}^r$	$L^m - \overline{M}^m$

Table 4 continued

	(7) Investment sector	(8) Foreign trade	(9) Government	(10) Bank	(11) Central bank	(12) Foreign exchange stabilization fund
	i	f	g	b	c	s
1		p_1E_1	$(1-\eta)G_1$			
2	p_2I_2	p_2E_2	$(1-\eta)G_2$			
3			wN^g	wN^b		
4	$-H$					
5		$-rp_5^*F$				
6			$-T$			
7	$-\gamma_i\Pi$		$\eta(G_1+G_2)$			
8	$-\overline{B}^i$	$-\overline{B}^f$	$-\overline{B}^g$	$-\overline{A}^b$	$-\overline{A}^c$	
9	$p_b\delta B^i$	$p_b\delta B^f$	$p_b\delta B^g$	$p_b\delta B^b$	$p_b\delta B^b$	
10				δQ^b	δQ^b	
11		$r(D_\$^f - S_\$^f)$				$rD_\$^s$
12	$L^i - \overline{M}^i$			$L^b - \overline{M}^b$	$-\delta M^c$	$-\delta M^s$

account the inefficiency in government expenditure, and assume that the budget for public investment does not affect public spending based on tax revenue. Under these assumptions, if the amount of public investment and the inefficiency in government expenditure are denoted by ΔG_2 and η respectively, the government's consumption in directly purchasing capital goods is represented by $(1-\eta)\Delta G_2$ while $\eta\Delta G_2$ represents the government's transfer payment in the form of a firm subsidy that might be added to the firm's earnings surplus. In addition, it is assumed that β times $\eta\Delta G_2$ is used for the investment by firms.

Under these assumptions, the increased increments of the total demand for capital goods and consumption goods are given in Eqs. (5) and (6), respectively, where $p_1\Delta D_1^w$ is the increased increment of the total demand for consumption goods by the workers and $p_1\Delta D_1^e$ is the increased increment of the total demand for consumption goods by the executives.

$$p_2\Delta X_2 = \beta\eta\Delta G_2 + (1-\eta)\Delta G_2 \tag{5}$$
$$p_1\Delta X_1 = p_1\Delta D_1^w + p_1\Delta D_1^e \tag{6}$$

The increased demands by workers and executives are given in Eq. (7), where ΔW and $\Delta\Pi$ are the increased incomes of workers and executives, respectively; these are given in Eq. (8), as the sum in each of rows 3 and 7 is zero.

$$p_1\Delta D_1^w = c_w(1-t_w)\Delta W,\ p_1\Delta D_1^e = c_e(1-t_e)(1-t_m)\alpha\Delta\Pi \tag{7}$$
$$\Delta W = wa_{31}\Delta X_1 + wa_{32}\Delta X_2,\ \Delta\Pi = mc_1\Delta X_1 + mc_2\Delta X_2 + \eta\Delta G_2 \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6), the increased demand of consumption goods is given in Eq. (9).

$$\begin{gathered} p_1\Delta X_1 = \frac{b_2'}{1-b_1}p_2\Delta X_2 \\ \text{where,}\quad b_1 = c_w(1-t_w)\frac{w}{p_1}a_{31} + c_e(1-t_e)(1-t_m)\alpha m\frac{c_1}{p_1} \\ b_2' = c_w(1-t_w)\frac{w}{p_2}a_{32} + c_e(1-t_e)(1-t_m)\alpha\left\{m\frac{c_2}{p_2} + \frac{\eta}{1-(1-\beta)\eta}\right\} \end{gathered} \tag{9}$$

Equation (9) represents the increased demand in the consumption goods industry that is induced by the increase in demand in the capital goods industry, which in turn stems from public investment.

The increase in GDP on account of public investment is calculated as the sum of the increased demand in both the consumption and capital goods industries, minus the total payment for imported materials as given in Eq. (10), where μ_1 and μ_2 are the ratios of the cost of imported materials to the prices of consumption goods and capital goods, respectively.

$$\Delta Y = \left\{(1-\mu_1)\frac{b_2'}{1-b_1} + (1-\mu_2)\right\} p_2 \Delta X_2,$$
$$\text{where} \quad \mu_1 = r\frac{p_5^*}{p_1}a_{51}, \mu_2 = r\frac{p_5^*}{p_2}a_{52} \tag{10}$$

Substituting Eq. (5) into Eq. (10), the multiplier of public investment is given in Eq. (11).

$$\frac{\Delta Y}{\Delta G_2} = \left\{(1-\mu_1)\frac{b_2'}{1-b_1} + (1-\mu_2)\right\}\{1-(1-\beta)\eta\} \tag{11}$$

5.2 Multiplier of Public Investment for Capital Goods Under a Balanced Budget Condition

When part of the funds required for public investment $\gamma \Delta G_2$, where $\gamma = 0 - 1$, is financed by tax revenue, government consumption other than the investment decreases by $\gamma \Delta G_2$, resulting in a decrease in the multiplier of public investment. Let the government consumption based on the tax revenue be efficiently conducted in the consumption goods market. Equations (6) and (11) are then revised as Eqs. (12) and (13) respectively.

$$p_1 X_1 = p_1 \Delta D_1^w + p_1 \Delta D_1^e - \gamma \Delta G_2 \tag{12}$$

$$\frac{\Delta Y}{\Delta G_2} = \left\{(1-\mu_1)\frac{b_2''}{1-b_1} + (1-\mu_2)\right\} 1-(1-\beta)\eta,$$
$$\text{where} \quad b'' = b' - \frac{\gamma}{\{1-(1-\beta)\eta\}} \tag{13}$$

5.3 Approximate Numerical Values of the Multiplier of Public Investment

Assuming typical values for the parameters included in Eq. (7), the numerical values of the multiplier of public investment can be estimated for various influential factors, such as the inefficiency in government expenditure, the proportion of profits distributed to executives, β in Eq. (11), and γ in Eq. (13). The assumed parameter values are set to be essentially the same as those employed in Morishima's analysis so that the calculated multiplier can be easily compared with that of Morishima, which corresponds to the case of 0% inefficiency (see Morishima 1984).

Let the workers' marginal propensity to consume (c_w) be 0.9 and the executives' marginal propensity to consume (c_e) be 0.4. The income tax rate is assumed to be 20% for both workers and executives. Both the indirect-tax rate on consumption goods and that on capital goods are 5% (i.e., $t_1 = t_2 = 0.05$), while the rate of mark-up (m) is 30%. Supposing that the capital goods industry is more labor-intensive than the consumption goods industry, let us assume that the wages per unit of output in the capital goods industry amount to 50% of the price of the product, while the same figure is only 40% in the consumption goods industry (i.e., $wa_{31}/p_1 = 0.4$ and $wa_{32}/p_2 = 0.5$). Given these numerical values, we can calculate both c_1/p_1 and c_2/p_2 as being 0.73.

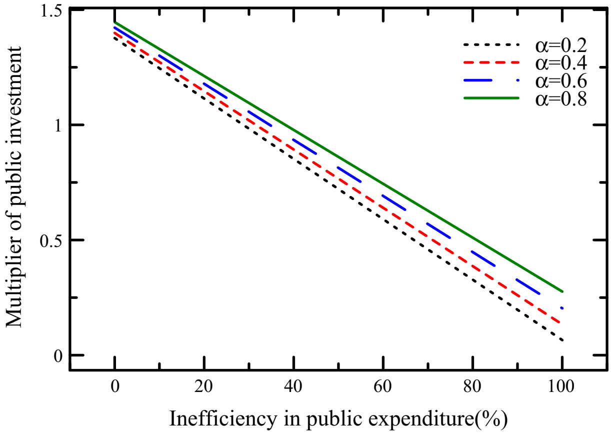

Fig. 7 Influences of inefficiency in government expenditure and the proportion of profits directed to executive compensation; $\beta = 0$

In addition, let each industry's marginal propensity to import with respect to its output be 10% (i.e., $\mu_1 = \mu_2 = 0.1$).

Given these numerical values (see Morishima 1984), we can calculate the multiplier of public investment for various values of its influential factors.

Figure 7 shows the multiplier of public investment as a function of the degree of inefficiency in government expenditure for various values of the proportion of profits to be distributed to executives α, assuming that β equals zero.

Let us first calculate the multiplier of public investment using Eq. (11) for the case where all required funds are financed by the government.

Figure 7 shows that the multiplier of public investment is greatly affected by the degree of inefficiency in government expenditure. The multiplier can be <1 if the inefficiency in government expenditure is 30% or higher. The proportion of profits to be distributed to executives α also affects the multiplier, but the degree of its influence is not overly large. The reason why the multiplier increases with an increase in α is that the part of the surplus earnings of a firm distributed to the executives increases the total demand for consumption goods from the executives.

It is noted in Fig. 7 that the multiplier is around 1.4 when the inefficiency in government expenditure is zero; this value is similar to that in Morishima's analysis, and is much smaller than the inverse of the marginal propensity to save. This is because the net influence of investment in GDP is reduced on account of the payment of taxes, the accumulation of surplus earnings by firms, and payments for imported materials. However, these factors cannot on their own account for a multiplier that is <1. We also note that in the case of Morishima's analysis, wherever the inefficiency in public investment is not taken into account, the multiplier of public investment in the capital goods industry is the same as the multiplier of investment made by the firms, as far as investment in the capital goods industry is concerned. In other words, there is no difference between the investment in the capital goods industry made by the government versus that made by private firms.

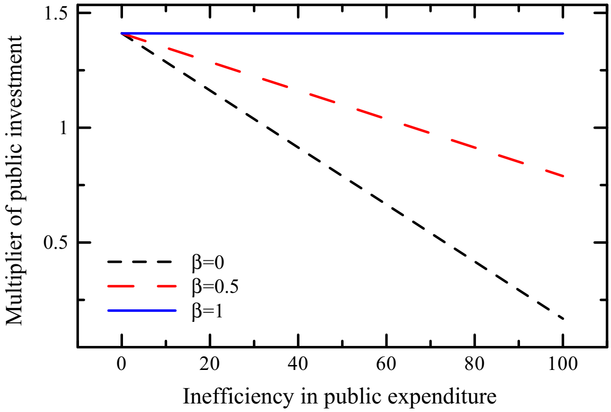

Fig. 8 Influence of the inefficiency in government expenditure on the multiplier of public investment and the influence of β in Eq. (13); $\alpha = 0.5$

However, it is considered reasonable to consider that there must be great difference between investment by the government and that by private firms; this will be discussed in the next section.

In the case that β times of the funds distributed to the firms by public investment in the form of a firm subsidy is used for firms' investments, the multiplier becomes larger with an increase in the value of β (Fig. 8); additionally, if $\beta = 1$, the influence of the inefficiency in government expenditure disappears. This is because when $\beta = 1$, all of the funds supplied by public investment—including that distributed to firms—are used for investment in the capital goods industry.

When the funds required for public investment are to some extent financed by tax revenue, the multiplier is greatly reduced (Fig. 9). This result suggests that the main reason for the occurrence of a negative multiplier is that the funds for public investment are not financed 100% by debt; rather, they are financed by tax revenue to some extent.

The calculated multipliers in Figs. 7, 8, and 9 appear to be smaller than those obtained in the agent-based simulation described in Sect. 4. This is mainly owing to the selection of parameter values and the assumptions made in the mathematical analysis. In the equation-based analysis, we assumed that the prices of consumption goods and capital goods are each unique and constant, and all economic indicators are in equilibrium. Moreover, the dynamic behaviors of the system, such as business cycle, time-dependent funds circulation, and interactions between agents, are not taken into account. Therefore, it is essentially impossible to obtain quantitatively same results for both approaches.

Regarding the selection of parameter values, however, much larger multiplier values can be obtained by adjusting these parameters in the mathematical analysis. Figure 10 and Fig. 11 are the calculated results corresponding to Figs. 7 and 9, respectively, when parameter values are adjusted to increase the consumer consumption. This adjustment is conducted by increasing the labor share, increasing the executive's marginal propensity to consume and decreasing the worker's rate of income tax, namely increasing c_e from 0.4 to 0.9, reducing t_w from 0.2 to 0.1 and increasing wa_{31}/p_1 and wa_{32}/p_2 to 0.8 in Eq. (9).

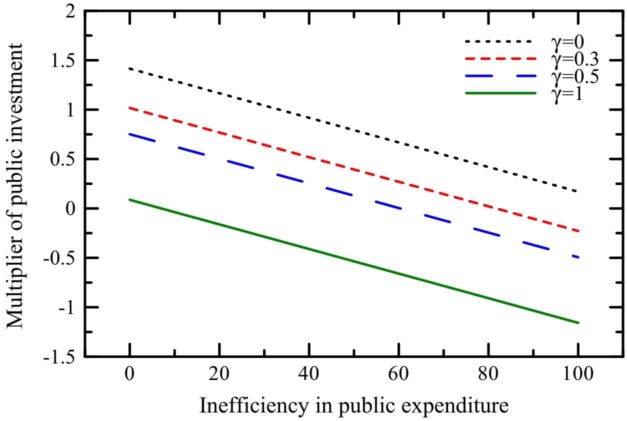

Fig. 9 Multiplier as a function of inefficiency in government expenditure, for various degrees of balanced-budget conditions (i.e., $\gamma = 1$: 100% balanced budget, $\gamma = 0$: 100% debt financing; $\alpha = 0.5$, and $\beta = 0$)

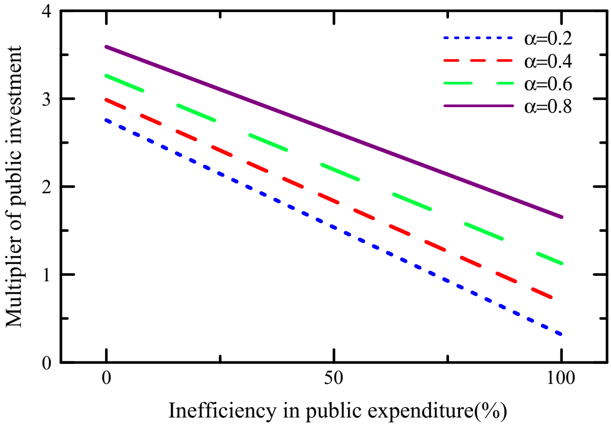

Fig. 10 Influence of inefficiency in public spending and the proportion of profits directed to executive compensation; $\beta = 0$, when adjusting parameter values toward increasing consumer consumption

It is noted that we can obtain much larger multiplier values in the equation-based analysis by adjusting parameter values toward increasing consumer consumption. It is also noticeable that the negative relationship between the multiplier and inefficiency in government expenditure is consistently obtained, regardless of the parameter values.

6 Discussion

6.1 Why Does the Multiplier Become So Small and What is the Source of Inefficiency in Government Expenditure in Actual Systems?

The results of the ABM approach (Figs. 3, 4, 5, 6) and the mathematical model approach (Figs. 7, 8, 9) in the present study consistently assert that the multiplier

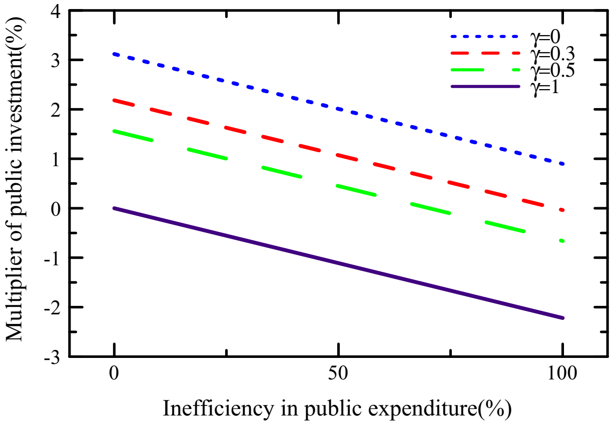

Fig. 11 Multiplier as a function of inefficiency in government expenditure, for various degrees of balanced-budget conditions (γ), when adjusting parameter values toward increasing consumer consumption ($\alpha = 0.5$, and $\beta = 0$)

of public investment is greatly affected by inefficiency in government expenditure. The inefficiency in public expenditure is defined in the present study as the ratio of government expenditure for firm subsidies to the total amount of public spending. As explained in Fig. 4, a part of the funds thus transferred to the firms as a result of inefficient government expenditure flows into the bank in the form of increasing firms' deposit or decreasing the loan balance, reducing the money stock in the market and thus the multiplier. Therefore, the direct answer to the question of why the multiplier can fall below 1 is that the influence of public investment on the economy is greatly deteriorated by inefficient government expenditure, because the funds transferred to firms due to inefficient government expenditure are likely to be saved in the bank as surplus earnings, and not be used to finance economic activities, decreasing the money stock in the market.

Another question then arises: why is government expenditure likely to be inefficient? As mentioned in Sect. 5, there must be a great difference between the government and private firms, in terms of their investment. A big difference between them is thought to be that the government has less incentive to minimize its cost than private firms. The payment made by the government to purchase goods or services is likely to exceed its actual economic value. The difference between them is something like a transfer payment to the firms or a firm subsidy that bears no restriction on its use. This increases the degree of inefficiency in government expenditure and decreases the multiplier, as indicated in Fig. 7.

Transfer payments to firms (such as firm subsidies) are made in a variety of ways under the mask of economic policy packages such as industrial promotion and the protection of domestic industry. These include direct transfer payments to firms for specific economic activities, in the case of the privatization of a government's specific function; the government's selling of public assets to firms at a price lower than the market value, where the difference corresponds to the firm subsidy in the present study; preferential treatment by the government toward a specific firm or business field (i.e., so-called rent-seeking); and the public payment of wages or rewards for activities or businesses that have little or no economic value, including firms' indirect payments to individuals or other firms from subsidized funds.

These kinds of transfer payments are not likely to be criticized, as they are often conducted nominally as policies by which to promote economy or welfare. Nonetheless, it should be noted that if the funds paid by the government to the firms or organizations are larger than the market value of their economic activities, then the difference will correspond to the firm subsidy in the present study, and it will cause inefficiency in government expenditure.

Because the government's role includes the provision of social welfare, some extent of inefficiency in government expenditure is indispensable and must be allowed. What the present study indicates is that if the inefficiency in government expenditure becomes too large, it greatly deteriorates the economy and much effort will be required to keep the inefficiency as low as possible from the view point of promoting the economy.

The inefficiency in government expenditure as defined in the present study can hardly be verified in an actual economy, according to government accounting data, because it would be difficult, after all, to compare government payments to corresponding market values. It is well known, however, that public investment often makes a very small economic impact. In addition, the literature asserts that the multiplier of public investment in the actual economy often has a very small value—sometimes <1. The results of the present study indicate that these facts can be effectively understood by introducing the idea of inefficiency in government expenditure. Such inefficiency is considered the main reason why multipliers are often so small, compared with the value expected by virtue of the marginal propensity to consume.

Based on the aforementioned discussion, we can say that the influence of inefficiency in government expenditure on the multiplier of public investment offers important real-world implications. It may explain, for example, why the repetition of public investment in Japan since the collapse of the bubble economy in 1989 has had little influence in stimulating the Japanese economy, even as the balance of government bonds there has increased. It may also explain why small-government policies in the United States and the United Kingdom in the 1980 s were effective. It may also explain why the multiplier of public investment is smaller than that of government consumption, which was pointed out by Perroti (2004); i.e., compared with the case of steadily operating government consumption, the government has less incentive to minimize costs in the case of public investment because it may be that public investment is likely to be subject to political pressure.

6.2 Reason for the Quantitative Difference in the Calculated Multipliers Between ABM and Mathematical Model Approaches

The ABM and mathematical model approaches consistently show that the multiplier strongly depends on the inefficiency in government expenditure. The two approaches are thus in good agreement qualitatively. From the viewpoint of the quantitative agreement of the two approaches, however, the multiplier obtained in the ABM approach is much larger than that obtained in the mathematical approach. The basic reason for this difference is the differences in the preconditions of the model and the selection of parameter values. In the case of ABM, macro phenomena or values of state variables such as the fund circulation, business cycle, prices of products, and demand and supply of products are endogenously and dynamically determined as a result of interactions between objects in the system. The multiplier is affected by these factors as well as parameter values. In the case of the mathematical model approach, fund circulation is assumed to be in equilibrium and independent of time and the price of the product is exogenously determined by the rate of mark-up. In addition, in the present study, parameter values are set to be the same as those assumed in Morishima's analysis. If parameter values such as the rate of mark-up and the marginal propensity to consume are changed in the mathematical model approach, it was found that a multiplier >2 can also be obtained. Similarly, the calculated multiplier could be smaller in the ABM approach if parameter values are adjusted.

There thus remains much work to be done for the two approaches to quantitatively agree, which is not the aim of the present study. The aim of the present study is to clarify why the multiplier of public investment can be smaller than 1 or even negative. In this respect, what is important in the present study is that the results of the two approaches consistently show that the multiplier decreases greatly with an increase in the inefficiency in government expenditure and that the multiplier can be <1 when the inefficiency is large enough.

6.3 Indispensable System Structure for Reproducing a Multiplier That has a Value <1

The agent-based model used in the present study comprises 150 consumers, 37 producers, a bank and a government, the system structure of which is the same as that of the model that reproduces the positive effect of tax reductions on income and corporate tax. Although this model may appear overly complicated to some, we believe that this system structure is almost indispensable for a model including a government role. This is because it also includes essential structures for reproducing the fundamental behavior of the macroeconomic system, such as fund circulation, price equilibrium, supply chain, the endogenous occurrence of firm investment as well as bank financing, and business cycles, in addition to the positive effect of tax reduction. Here the system structure is defined as a set of agents and other including objects in the system, their behavioral rules and the field where they develop their activities. For example, executive compensation and inefficiency in government expenditure are necessary to reproduce the positive effect of corporate tax reduction, and the positive effect of tax cuts in both income tax and corporate tax, respectively. The set of behavioral rules for the production and pricing strategies of producers, as well as those regarding the buying strategies of consumers, are essentially indispensable for reproducing endogenous price equilibrium. Moreover, the bonus is thought to be essential to reproducing stable fund circulation, at least in the case of a limited number of agents (please see Ogibayashi and Takashima 2009, 2013, 2014).

We then ask, what is the system structure indispensable to reproduce a multiplier with a value of <1? Answering this question might mean simplifying the model by removing factors one by one. Strictly speaking, this is future work. In the present study, however, it is almost possible to answer this question, because the results obtained from ABM and mathematical approaches agreed qualitatively, as they both consistently show a negative relationship between the multiplier and inefficiency in government expenditure.

Therefore, the indispensable system structure for reproducing a multiplier with a value less than one is that which includes inefficient government expenditure, as well as most of the other factors included in the present model. Additionally, although the interest rate is believed to play a significant role in the macroeconomy, interest rate flexibility is not considered necessary. It should be noted that qualitatively, the same results were obtained using the ABM approach, in which the interest rate is assumed to be constant, and the mathematical model approach, in which the interest rate is not considered. This suggests that the effect of interest rate on the multiplier is negligible compared with the effect of inefficiency in government expenditure. We shall discuss the effect of interest rate in more detail in the next section.

6.4 Effect of Interest Rate Flexibility on the Multiplier of Public Investment

Do the arguments made in the present study hold if the interest rate changes in response to deposits?

In the present study, the interest rate is assumed to be constant at 3%. This section discusses the effect of interest rate flexibility on the multiplier of public investment. It should be considered whether the negative influence of the inefficiency in government expenditure on the multiplier could become smaller than that estimated in the present study, if the interest rate is assumed to decrease temporarily in response to an increase in deposits caused by government subsidy. The reduction in interest rate may prompt firms to borrow and take up new investment projects, thus causing the firm's increased deposits to re-enter circulation. This case corresponds to the β larger than 0 in Fig. 8. If β times the funds distributed to the firms in the form of a subsidy is consumed for additional investments, the multiplier increases alongside that of the value of β. Therefore, the question is whether with the flexible interest rate, this additional investment could consequently occur because of the decrease in interest rate caused by an increase in the firms' deposits.

The authors' answer to this question is NO. Even though the increase in firm deposits results in a decrease in interest rate, it might not prompt firms to make additional investments. Thus, the effect of interest rate flexibility on the multiplier of public investment is considered to be negligible compared with the effect of inefficiency in government expenditure, the reason for which is discussed below.

In the present study, it is assumed that firms decide to invest primarily on the basis of their total sales (i.e., demand) and secondarily based on Eq. (3), which represents the condition of cash flow. Because 50% of required funds for investment are assumed to be financed internally, the condition of Eq. (3) is fulfilled in most cases, and therefore the firm's decision on investment is mostly based on expected demand. Thus, the effect of interest rate flexibility on the multiplier of public investment is considered negligible in the present model. Subsequently, the next question arises: whether the behavioral rules on the firm's investment assumed in the present model are reasonable.

In general, firms decide to invest on the basis of expected return, which depends on the demand and interest rate. The effect of the interest rate is evaluated by either the discounted present value of the investment, which must be larger than the funds for investment, or the cash flow, which must be positive. Let us assume that required funds for investment A are all financed by the bank; equal repayment with interest plan is employed for n years, which is the same as the depreciation period of the equipment; and the value of expected return R is constant for n years. Then the conditions for the discounted present value of the investment and the cash flow are coincident and approximately represented by $R/A > (1+r)/n$. Let n be 10; then, the value of $(1+r)/n$ changes only 2% when r is increased from 0.01 to 0.03. As apparent from this example, the influence of interest rate on the decision to invest is negligible compared with the uncertainty of the expected future return. In other words, the decision to invest is primarily made on the basis of demand, and therefore, the effect of interest rate on this decision is considered to be negligible.

The argument stated above seems to be consistent with existing literature. According to questionnaire survey results presented by Oxford (Andrews 1940), around 80% of the survey respondent firms answered that the interest rate and other factors relating to the cost of borrowing money do not affect their decision making regarding investment. Haavelmo (1941) theoretically discussed the effect of interest rate on investment and came to similar conclusions.

Further evidence that the effect of interest rate on investment is negligible is the recent result of the quantitative monetary easing policy promoted by the central bank of Japan. This policy was conducted in an attempt to decrease interest rates, prompt firm investments and to increase consumer price. However, it is noted in the literature (Yoshikawa et al. 2015) that this policy does not have any influence on consumer price. This result suggests that the firm's decision making regarding investment is mostly based on demand and interest rate has a relatively minor influence.

From these results, it appears that the most essential factor affecting firm investment is the expected return based on demand. The effect of interest rate on investment decision-making is negligible compared with the uncertainty of expected future return. Therefore, the effect of a flexible interest rate on the firm's investment is considered to be negligible.

7 Conclusion

The present study analyzed the influence of the inefficiency in government expenditure on the multiplier of public investment, through the use of both ABM and a mathematical derivation of the equation for the multiplier of public investment based on an economic linkage table. The results of both approaches indicate that the multiplier of public investment is strongly affected by inefficiency in government expenditure, which is defined as the ratio of firm subsidies to total government expenditure. In addition, the multiplier of public investment becomes <1 when inefficiency is sufficiently large and firms are reluctant to invest. In the case of a balanced-budget condition—where funds for public investment are financed by fiscal tax revenue to some extent—the multiplier can be negative when government expenditures are inefficiently managed.

It is thought that the source of this inefficiency in government expenditure comes from the likelihood of the government having less incentive than firms to minimize costs, and that in some cases, public investment is likely to be subject to political pressure; such conditions may lead to irrational public spending. If government spending is more effectively conducted and costs are minimized while political pressure is resisted, the multiplier of public investment could be expected to be much larger; under such circumstances, it would be possible for public investment to be effective in promoting the economy, while keeping government debt to low levels.

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