

Model Structure of Agent-Based Artificial Economic System Responsible for Reproducing Fundamental Economic Behavior of Goods Market

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Abstract. Validation has been an important issue in using the ABM approach. It has been pointed out that deriving the necessary conditions for reproducing specific macro behavior is difficult due to the functional complexity of ABMs. However, based on the authors' experience with ABMs, we believe it is possible to define the necessary conditions for reproducing each macro behavior by using the structure of the system to express the input conditions. In the present study, a series of computer experiments are conducted to verify this idea. The study analyzes business cycles and the effect of tax reductions on GDP as examples of fundamental macro behaviors of economic systems. The results indicate that the most essential model structures for reproducing business cycles and the effects of tax reduction are credit creation for investment and factors relating to the efficiency of the government's, household's, and firms' expenditures, respectively.

Keywords: Agent-based modeling, Model structure, Business cycle.

1 Introduction

Agent-based modeling is a bottom-up modeling method in which we view artificial, computer generated societies as laboratories where we attempt to grow certain social structures. The purpose of these models is to discover the fundamental local or micro mechanisms that generate macroscopic social structures and collective behaviors [1]. Although agent-based modeling is a promising methodology that can deal with heterogeneity, individual agents' bounded rationality, and non-equilibrium dynamics in social systems, validation still proves to be a significant issue. As pointed out in the literature [2], one typical criticism by economists could be stated as follows, "you have presented one set of behavioral rules to explain your chosen phenomenon, but there must be many such sets which produce the same result, so how do you know yours is correct?" Some economists even go so far as to imply that it is excessively easy to construct an agent-based model (ABM) that produces desired phenomena. As argued by Marks [3], the problem behind this criticism is the functional complexity inherent in the ABM. It has also been argued that macro behaviors may be insensitive to many micro variables; and, as a result, it would be difficult to derive the necessary

conditions for the model to exhibit specific macro behaviors [3]. The severity of this problem increases when the model is described with greater detail and realism as this requires more variables and greater degrees of freedom [3]. For this reason, the model should be as simple as possible, and even then, it would be difficult to achieve quantitative predictions.

When input conditions are expressed by specific values of micro variables or parameters, there is a great deal of freedom. However, it should also be noted that the freedom of input conditions decreases if they are expressed by the system structure of the model (i.e., model structure) [4]. Here, the model structure includes the types of agents, the type of field (such as the market in which agents develop their activities), and the agent's behavioral rules. Consequently, it would be considered possible to specify the necessary conditions to reproduce the specific macro behavior. This is consistent with the argument of Ormerod [2] who pointed out that the current method used to build ABMs is a process of discovering the behavioral rules for agents that appear to be consistent with the phenomena we observe.

In this context, we believe that, although the model should be as simple as possible (based on the KISS Principle [5]), it is also important to consider all of the factors required to reproduce the desired phenomena. That is, the model structure should be the same as, or similar to, the real system in order for the characteristics to emerge as they do in the real world. The factors essential for reproducing the desired characteristics of the system can be discovered by running a series of computer experiments in which only one constituent factor of the model is changed at a time [4].

Although a number of ABM research studies have focused on macroeconomic aspects, these studies have not fully clarified the structural factors necessary for their reproduction.

Motivated by this deficiency, the authors have constructed a simple, artificial economic model consisting of consumers, three types of producers, a bank, and a government (some of which were reported in previous studies [4,6,7]).

In the present study, some additional simulations are conducted to clarify the model structure necessary for reproducing business cycles and the changes in GDP caused by a tax reduction (which were taken as examples of fundamental macro behaviors in a goods market). A series of simulation experiments are systematically conducted, changing the input conditions one by one. The study focuses on finding the model structure necessary to reproduce the above mentioned macroeconomic phenomena.

1.1 Outline of model

The ABM of the artificial economic system in the present study includes consumers, producers, a bank, and a government as autonomous decision-making agents. Consumers and producers are each divided into three types of agents, as shown in Table 1. Markets are also divided into three types: goods, stock, and labor. Each agent is heterogeneous in its state variables as well as in the other parameters included in their action rules.

Table 1. Outline of agents and their action rules.

Agent	Type	Output to be supplied	Product type to purchase	Outline of action rules	
Consumer	Worker	The labor force for firms	Consumption goods	Consumers work and obtain wage at producer, bank, or government, pay tax, and purchase consumption goods. Income is divided into the money for consumption and deposit by the Keynesian consumption function. Purchasing consumption goods is performed according to the utility which each consumer holds uniquely. Consumers transact in stock market, aiming to increase their assets, when stock market is	
	Executive	Management for firms			
	Public worker	The labor force for government			
Enterprise				Enterprises employ consumers, get profits from operating activities and pay wages and tax.	
	Producer	Retailer	Consumption goods	Consumption goods, Materials, Equipment	Producers supply and sell products in the goods market. Retailers and raw material producers decide both the amount and price of each class of product according to its state of amount of inventory. Moreover, they invest in equipment and expand production capacity according to demand.
		Raw material producer	Material goods	Consumption goods, Equipment	
		Equipment manufacturer	Equipment	-	The equipment manufacturers produces equipment in line with the requirements of the retailers and raw material
	Bank	Bank	The funds for producers	-	The bank keeps the surplus money of other agents in their respective bank accounts, and lends money.
Government	Government	Redistribution of wealth	Consumption goods	The government collects tax from other agents, pays wage to public worker and spends remaining money on expenditure of public money.	

1.2 Sequence of actions

The set of actions for each agent is comprised of period-based units, where one period is assumed to correspond to one month in the real system. During each period, agents act according to a sequence of eight steps. At the end of the sequence for each period, a GDP value is calculated based on an input/output table obtained by summing each agent's account data. The eight steps dictating the agents' actions are as follows:

1. Agents pay any unpaid tax from the previous period. After paying taxes, agents create a budget plan for consumption, paying wages, or public spending.
2. Raw material producers decide on the quantity and price of products to be produced, produce several types of raw materials, and supply these to the goods market.
3. Retailers decide on the quantity and price of products to be produced, purchase raw materials in the material goods market, produce several types of consumption goods, and supply these products to the consumption goods market.
4. Consumers, retailers, raw material producers, and the government purchase products in the consumption goods market.
5. Each firm pays wages to employees and executive compensation to the executives while the government pays wages to public workers.
6. Retailers and raw material producers consider expanding production capacity based on total sales in the previous periods, and, if necessary, they decide to invest in expansion by either buying new equipment from the equipment manufacturer or employing a new worker.
7. When a stock market is included in the model, consumers buy or sell stocks aiming to increase their financial assets.

8. Each agent settles its accounts using the double-entry bookkeeping method. They calculate their income and profit for the current term, and then determine the amount of tax to be paid based on these figures.

1.3 Outline of agent's decision-making rules

1.3.1 Behavioral rules of consumers

Consumers create a budget for consumption E_b^t . This budget is calculated by adding after-tax income $I^t (1-r_{i_tax})$ which represents the Keynesian consumption function [8]), to their bank deposit D^t multiplied by a withdrawal ratio r_{wd} at each fiscal period t . The formula for the budget is shown in Equation (1). Here, r_{i_tax} is the income tax rate, a , is the consumer's basic consumption, and b is the marginal propensity to consume as per the Keynesian consumption function. The withdrawal ratio r_{wd} is selected randomly for each agent during each period.

$$E_b^t = a + bI^t(1 - r_{i_tax}) + r_{wd}^t D^t \quad (1)$$

When purchasing products in the consumption market, consumers select goods based on their utility and affordability (as determined by the utility function for each class of products and the agent's budget constraint, respectively). Moreover, when a stock market is included in the model as an experimental level in order to analyze the reproducibility of business cycles, consumers buy or sell stocks aiming to increase their financial assets. Please refer to the authors' previous study in which consumers' action rules in the stock market are described in detail [7].

1.3.2 Behavioral rules of producers

The retailers and raw material producers both decide the quantity and price of their product at the beginning of each period. The price of each product is increased or decreased depending on the amount of goods they held in stock at the end of previous period. The quantity to be produced is decided in such a way that the probability of being out of stock must be less than 5%; this is estimated based on total sales from the last 10 periods.

The production capacity Y is defined by the Cobb–Douglas function [8] (as shown in Equation (2)) where K is the number of units of capital equipment, L is the number of employees, and α is assumed to be 0.25. In addition, A is a bounded proportionality constant that is randomly assigned to each producer. It is assumed that this value is unique to each producer and represents its technical capability.

$$Y_i(K, L) = A_i K^\alpha L^{1-\alpha} \quad (2)$$

Retailers and raw material producers initially have one unit of equipment and a specified number of employees. They will invest in order to increase their production capacity after they have passed a determined number of periods producing at maximum capacity. They decide to invest based on expected financial merit obtained by either buying a piece of equipment from the equipment manufacturer or employing a

new worker from the labor market (when a labor market is included in the model as an experimental level).

When investing in equipment, they may finance the funds by borrowing from the bank, issuing new shares in the stock market, using their own internal funds, or using some combination thereof. The funds financed by the bank are repaid with interest in equal sized payments each period for a constant number of consecutive periods. An upper limit is placed on total investment so that, during the repayment period, additional investment will not be allowed. The equipment manufacturer produces equipment in accordance with the requirements of retailers and raw material producers as long as it is within their capacity. In the present study, the price of equipment is assumed to be constant. Please refer to the authors' previous study in which the decision-making rules for investment as well as for financing are described [7].

One executive and several workers are initially assigned to each of the producer agents. The producers pay wages to workers and wages plus executive compensation to the executive in each period. The executive compensation comprises a salary, a bonus, and long-term incentives. Wages comprise a fixed salary (randomly assigned to each employee between a lower and an upper limit) and a bonus when the producer's profit is positive.

1.3.3 Behavioral rules of bank

The bank lends money in the form of long-term loans to producers (in line with their demands for investment), charging a 3% interest rate. The bank also lends money to producers in the form of short-term loans so that they may meet their requirements when their working capital to pay fixed wages and/or purchase raw materials becomes sufficiently depleted. In the present study, the bank is initially given a very large quantity of funds so that there is no limitation on lending to producers, except in the case where long-term loan payments are not fulfilled during the repayment period.

1.3.4 Behavioral rules of government

The government collects corporate and income taxes, pays wages to public employees, and uses the surplus funds for public expenditure as dictated by their expenditure policy. Public employees' wages are calculated in each fiscal period so that they are equal to the average income of private employees.

Concerning expenditure policies, the study tests market purchasing, firm subsidies, and combinations thereof. Market purchasing is an extremely efficient form of public expenditure in which the government directly purchases goods at the market price. This policy is akin to the government placing job orders with firms, in a completely competitive situation, at the market price. Firm subsidies are an extremely inefficient form of public expenditure in which the government distributes funds to producers, without any limitations on their use. Most of the funds distributed could be transferred to the bank account without being used in the market. This policy is akin to the government placing job orders at a value far above the market price or paying money for jobs that have no economic value.

2 Simulation Conditions

The simulation conditions as experimental levels are divided into two categories: an analysis of the reproducibility of periodic changes in GDP (i.e., business cycles) and an analysis of the reproducibility of the effects of tax reductions on GDP.

In the former experiment, producers' decision making processes regarding investment in equipment (including the case where they do not invest) and the means of financing said equipment, as well as the types of markets are manipulated as input conditions in order to find the necessary model structure for reproducing periodic change in GDP (i.e., a business cycle). The periodic changes in consumers' wages and the amount of money spent investing in equipment are also analyzed. In the latter experiment, the types of agents included in the model system (including executives) and their behavioral rules relating to the efficiency of government', producers', and consumers' expenditures are all changed as input conditions so that the influence of tax reductions on GDP may be analyzed.

A series of simulation experiments are systematically conducted, changing the factors relating to the model structure (such as the type of agents, their behavioral rules, and the type of market) one by one. The simulation conditions for the experiment are shown in Table 2.

Table 2. Simulation conditions for the experiment in which factors relating to the model structure are changed as input conditions.

		Structure of basic model	Analysis of reproducing the periodic change of GDP			Analysis of reproducing the influence of tax reduction
			Analysis of investment rules	Analysis of financing rules	Analysis of MEC model	
Agent	Government	Without	Without			With
	Others	With	With			With
Rules of producers	The decision-making rule of equipment investment	Based on demand	No investment / Fixed interval	Based on demand	Based on an internal rate of return	Based on demand
	The rule of financing	Loan and internal funds	Loan	Using internal funds / The issuance of stock	Using internal funds	Bank financing / Loan and internal funds
	The rule of executive compensation	Without	Without			With / Without
	The deletion of equipment	Without	Without		With	Without
	The price of equipment	Fixed	Fixed		Variance	Fixed
	The upper limit on the number of loans	Limited(1)	Limited(1)		Unlimited	Limited(1) / Unlimited(3)
Rules of consumers	The rule of withdrawal deposit	With	With			With / Without
Rules of government	Taxation	Without	Without			With
	Inefficiency of government expenditure	Without	Without			With
Market	Goods market	With	With			With
	Stock market	Without	Without	Without / With	Without	Without
	Labor market	Without	Without			With / Without

3 Simulation Results

3.1 The necessary model structure for reproducing business cycles

As shown in Fig.1, the cyclical changes in the average price of consumption goods, average consumer income, and GDP are reproduced by the simulation under the base

model conditions. The necessary funds for investment are all financed from the bank with constant repayment periods. It should be noted that these three macro indicators shows synchronized movement.

The business cycle mechanism reproduced by the base model is summarized as follows: In the beginning of the booming stage, some of the firms with strong sales decide to invest in equipment. This induces an increase in demand, wages, and investment at the aggregate level.

After the majority of producers have made their investments, the total amount of repayment per period becomes larger than the total amount of borrowing due to credit rationing. This induces a decrease in total sales, workers' wages, and investments, thus resulting in a recession. The details are presented in our previous studies [6].

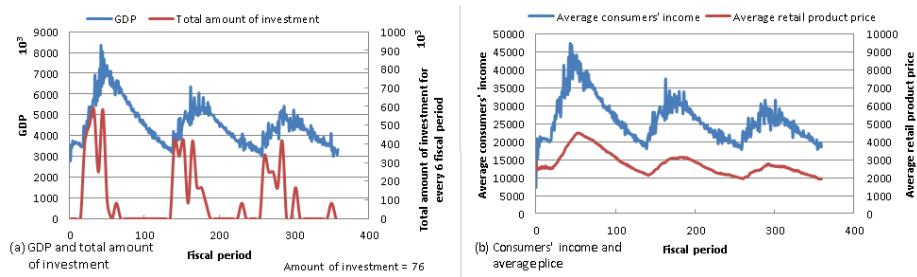


Fig. 1. The change in GDP and total amount of investment (a, left) and average consumer income and average retail price over time(b, right) under the conditions of the base model (bank financing and decision-making on investment on the basis of demand).

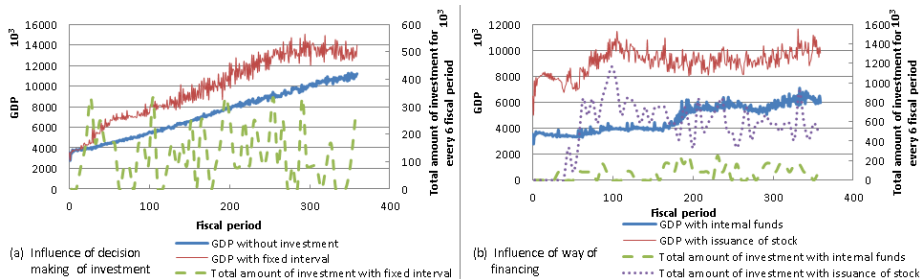


Fig. 2. Influence of investment decision making (a, left) and influence of financing means (b, right) on GDP and total investment.

When we assume that producers either do not invest (i.e., there is no debt), or that they invest randomly, with no regard to total sales, then there is no periodic change in GDP (as shown in Fig.2 (a)). Therefore, it might be concluded that the model must incorporate endogenous capital investment decision making (dictated by demand) in order to reproduce business cycles.

Financing from the bank (i.e., the existence of loans) is considered to be another important condition for reproducing business cycles. Fig.2 (b) shows the change in GDP when investment is financed either by the internal funds alone or by a combina-

tion of internal funds and the issuance of new shares in the stock market. In both cases, we can see the fluctuations in GDP and in the number of investments, but periodic changes in GDP (business cycles) do not occur. This is because, in both cases, there are almost no definite restrictions for conducting additional investment. When only internal funds are used, GDP shows slight cyclical variations (as shown in Fig.2), but this tendency is far less clear than that of the bank financing case. This is because some firms must wait several periods after enough funds have been raised in order to invest, but not all of the firms do. In addition, clear periodic change in GDP occurs in both cases when the bank financing rule is added (see Fig.2).

Therefore, in an ABM featuring producers' production and pricing activities as well as consumers' buying and working activities, it is reasonable that the most important conditions for reproducing business cycles would be the inclusion of bank financing and investment in the model structure.

On the other hand, Keynes proposed that the marginal efficiency of capital (MEC) is the primary determinant of the business cycle [8]. This, in turn, implies that the internal rate of return is the essential factor for creating business cycles. Based on this idea, an additional experiment is conducted in which producers decide to invest when the internal rate of return is expected to be greater than the current interest rate. Here, the internal rate of return is calculated using the expected value of the investment's marginal productivity, the price of the product, and the operating ratio of the equipment. The life of the equipment is assumed to be 60 and the price of the equipment is assumed to be $EP^t + I = EP^t(1 + 0.1(O^t/Y))$, where EP^t is the price of the equipment in period t , O^t is the amount of orders received in period t , and Y is the production capacity of the equipment manufacturer. However, cyclical change in GDP does not emerge in the simulation when bank financing is excluded from the model. The primary reason for this is that there is little to no change in the aggregate capacity of supply. The decreases in production capacity suffered by some producers due to the scrapping of equipment are balanced out by the surpluses of others. As such, without bank financing, variation in production capacity due to the scrapping of or investment in equipment cannot, by itself, influence the price of the retail product, and hence the expected return. Therefore, marginal efficiency of capital is not considered to be a major factor for generating business cycles when there is any degree of surplus in the aggregate production capacity.

3.2 The influence of a reduction in income and corporate taxes on GDP

In addition to the factors included in the base model (where the types of agents included are private and public workers as consumers; retailers, raw material producers, and equipment manufacturers as producers; a bank; and a government) each agent's behavioral rules regarding consumption are changed so that their influence on the relationship between the tax rate and GDP may be analyzed. The base level tax rate is initially set at 30%. In order to analyze the influence of a tax reduction on GDP, the tax rate is reduced from its initial level to 20% or 10% after 100 periods, while the average GDP over 360 periods is employed as the macro indicator.

The calculated relationship between the income tax rate and GDP is shown in Fig.4 (a). It should be noted that the negative correlation between the income tax rate and GDP is only reproduced when some inefficiency exists in government expenditure. It is also found that the level of inefficiency at which the correlation changes from positive to negative decreases when the effective marginal rate of consumption (which is dependent on the withdrawal ratio on bank deposits and the existence of executives) is increased.

If government expenditure is sufficiently inefficient, the negative correlation between the income tax rate and GDP is reproduced regardless of credit rationing (i.e., the upper limit on the number of loans), the existence of executive compensation, the usage of internal funds for investment, consumers' withdrawal ratio on bank deposits, or the labor market.

Consequently, it seems that the most important factor for reproducing the negative correlation between GDP and the income tax rate is the inefficiency of government expenditure. If government expenditure is 100% efficient, GDP increases even when income taxes increase. The reason for this tendency is that as the efficiency of government expenditure corresponds to the government's marginal propensity to consume. If the efficiency of government expenditure is larger than the consumers' marginal propensity to consume, some of the consumers' money to be deposited in the bank account will be transferred to the government by taxation and then consumed in the market, leading to an increase in GDP with an increased tax rate.

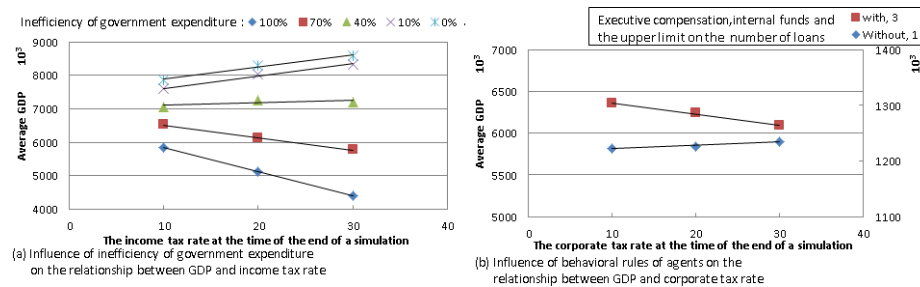


Fig. 3. Influence of behavioral rules (which are executive compensation, using internal funds, the upper limit of number of loans being three, and deposit withdrawal) on the relationship between GDP and income tax reduction (a, left) and corporate tax reduction (b, right)

However, the negative correlation between the corporate tax rate and GDP is not reproduced when only the inefficiency of government expenditure is accounted for. The negative correlation is only reproduced when executive compensation, the usage of internal funds for investment, and the inefficiency of government expenditure are all taken into account. The results also show that if the inefficiency of government expenditure is great enough, the negative correlation is reproduced regardless of the upper limit on the number of loans, the withdrawal ratio on consumers' bank accounts, or the existence of a labor market.

These factors might affect the critical level of efficiency at which the correlation changes from negative to positive.

Therefore, it seems that executive compensation, the usage of internal funds for investment, and the inefficiency of government expenditure are indispensable factors for the model to reproduce the negative correlation between the corporate tax rate and GDP. Although corporate tax reduction is known to reduce unemployment in the real system [9], the results show that the inclusion of a labor market in the model (which would account for unemployed workers) is not an indispensable factor in reproducing the negative correlation.

The reason for this tendency is that the surplus money from tax reduction promotes investment when the model accounts for the usage of internal funds, and directly increases demand when it accounts for executive compensation. If these two factors are not taken into account, the surplus money from the tax reduction is only transferred to the firms' bank account without increasing the market demand.

This finding suggests three things:

First, when input conditions are expressed by the model structure, it is possible to specify the necessary conditions to reproduce specific macro behavior. In addition, we can gain understanding of the underlying mechanisms that produce the specific macro behavior by discovering the necessary system structure for the model.

Second, corporate tax reduction increases GDP only when the government's effective marginal propensity to consume (expressed by the degree of efficiency in government expenditure) is smaller than that of aggregate firms'. That is, GDP is increased when producers receive surplus money from the tax reductions and effectively spend it in the market by means of investment or consumption by executives and workers.

Third, inefficiency of government expenditure harms the economy. The degree of inefficiency is defined as the ratio of firm subsidies to the total amount of public expenditure. In the actual system, inefficiencies might be caused by many factors such as public orders set above the market price, subsidies to firms in the industry, or rent seeking behavior [10].

4 Discussions: the validity of the model in ABM

As described in the introduction, the validity of the ABM has been widely criticized. It has been pointed out that, due to the functional complexity of the system in an ABM, one cannot assume that the factors that successfully reproduce the desired macro phenomena are necessary conditions.

On the other hand, the results of this study indicate that the necessary conditions for reproducing both business cycles and GDP reactions to tax reductions exist. Furthermore, these factors can be determined by running a series of computer experiments where each of the factors is changed one at a time. These necessary conditions are the factors of the model structure and they include the types of autonomous, decision making agents; their behavioral rules; and the types of markets or other fields where the agents develop their activities. Moreover, by revealing the necessary conditions for the model to reproduce the specific macro phenomenon, it is possible for us

to gain a better understanding of the mechanisms that drive the macro phenomenon in question. The reason for this is discussed below.

A system is a set of interacting objects and is defined as a proper relation of sets [11]. Social systems consist of such objects as autonomous decision makers (i.e., agents, such as individuals and firms) and the field where they develop their activities (such as markets and cities). The macro behaviors of social systems are determined by the actions of interacting agents who develop their activities on the field.

Therefore, the macro phenomena that emerge in the model will be similar to that of the actual system if the set of agents (including their behavioral rules), the set of fields (such as the market where they act), and the set of attributes ascribed to the various types of agents are similar to those in actual systems. In other words, if the factors in the model are quite different from those of the actual system, then the macro phenomena in question will not be reproduced. Just how similar the factors must be in order to reproduce the desired phenomenon depends on the phenomenon in question as well as the form of similarity (i.e., whether the model factors are reproducing the qualitative or quantitative characteristics of the system). As evidenced by the results of present study, the most important of these factors are the types of agents and their behavioral rules. The results indicate that if these important factors are significantly different from those in the actual systems, the desired macro phenomena will not emerge either on a qualitative or on a quantitative level.

On the other hand, the model system does not need to exactly mimic the actual system in terms of the number of agents or the parameters on their attributes in order to reproduce the phenomenon, because, as pointed out by Marks [3], macro behaviors are insensitive to these factors. However, this study has found that the macro behaviors are not insensitive to the types of agents and their behavioral rules. This suggests that it is possible to specify the necessary structure of the model system by a series of computer experiments, if the types of agents and their behavioral rules are taken into consideration as integral factors.

It should also be noted that the model structure that can reproduce the desired macro phenomena might not be unique, because the emergence of macro phenomena could be influenced by several factors. However, this does not contradict the validity of the model, because each of the factors corresponds to a certain mechanism which would not be unique even in the real systems. Moreover, the mechanism of emergence for each phenomenon can be discovered by accumulating the knowledge on the model structures necessary for the reproduction of that phenomenon.

5 Conclusion

1. In an ABM where producers' production and pricing activities, as well as consumers' buying and working activities are included, the necessary conditions for the model to reproduce business cycles are the inclusion of bank financing and producers' capital investment decisions based on demand.
2. In order to reproduce a positive multiplier on income tax reduction under the balanced budget condition, the model must include inefficient government expenditure.

Furthermore, it is indispensable that the model include executive compensation and the usage of internal funds for investment in addition to the inefficiency in government expenditure in order to reproduce a positive multiplier on corporate tax reduction.

3. These results indicate that the necessary conditions for reproducing each of the macro phenomena can be identified if the input conditions of the model are expressed by the model structure (such as the types of agents, their behavioral rules, and the types of market). The model structure that reproduces the desired macro phenomena might not be unique. This does not contradict the validity of the model, though, because factors responsible for the emergence of macro phenomena might not be unique in the actual system. The mechanism of emergence for each phenomenon can be discovered by accumulating the knowledge on the model structures necessary for the reproduction of that phenomenon.

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