

Analysis of the Influence of Expenditure Policies of Government on Macroeconomic behavior of an Agent-Based Artificial Economic System

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Abstract. An agent-based model of artificial economic system including government has been developed and the influences of public policies on GDP and related emergent behavior of macroeconomic phenomena have been analyzed. It was revealed that power law distributions emerge during simulation in some factors such as assets of agents and GDP increases with an increase in the ratio of efficient expenditure policy such as market purchasing, Average price increases or decreases when GDP increases or decreases, and the influence of tax rate on GDP depends on the way of public spending. Most of these results are found to be quite consistent with real data, if it is assumed that public spending includes more than 10% of inefficient factors.

Keywords: Agent-based modeling, computational economics, government, expenditure policy, GDP, tax rate, power law distribution.

1 Introduction

Agent-based modeling (ABM) is widely used in social simulation, because it is a class of modeling method used to explain or understand social phenomena via a bottom-up approach [1,2]. Application of ABM in macro-economic systems is one of the most important research fields, because macro-economic systems are very complex and relate to many social phenomena. ABM is expected to propose a new way to understand the mechanism of the behavior of complex macro economy. In principle, it might even be possible to utilize ABM to understand or evaluate the effectiveness of economic policies such as tax reduction and public spending, as pointed out in the literature [3].

The prior studies on ABM of macroeconomic systems are considered to be divided into two types, the research which mainly focuses on the emergence of macroeconomic phenomena and its mechanisms [4,5] and the research which mainly focuses on the development of the models of multiple-market economy as a whole, taking into account the structure of real economy in as detail as possible[6,7]. Although there has been a lot of research which focuses on various aspects, there has not been much research which has focused on the role of government and the influence of public policies on GDP. In a previous research[8], authors have

constructed a simple artificial economic model composed of consumers, three types of producers, and a bank, and revealed that this simple model reproduces fundamental economic behavior such as loose equilibrium in price, business cycle due to capital investment, influence of money supply on GDP, etc.

In this study, an agent based model of an artificial economic system including government has been developed based on the authors' previous model. Using the model, the influences of public policies on GDP and related emergent behavior of macro-economic phenomena has been analyzed. The obtained results are compared with those of a real system to check whether the model reproduces the real phenomena.

2 Simulation Model

The agent-based model of an artificial economic system in the present study is composed of consumers, producers, a bank and government as autonomous decision making agents as shown in equation (1). It is assumed that producers are divided into 3 types of agents, i.e. retailers who produce final products for consumers, wholesalers who produce and supply raw materials for retailers and an equipment maker who supplies equipment for production for other types of producers. Consumers are divided into private employees who work for a private company and public employees who work for the government. Each agent has state variables and rules of actions. Market also has state variables although it does not have rules of actions. State variables of each agent, related to cash and deposit, at the beginning of each fiscal period are given in equation (2).

$$\begin{aligned} Agent &= \{C, P, B, G\}, P = \{R, W, E\} \\ C &: \text{Consumer}, P : \text{Producer}, B : \text{Bank}, G : \text{Government} \\ R &: \text{Retailer}, W : \text{Wholesaler}, E : \text{Equipment - maker} \end{aligned} \quad (1)$$

$$\begin{aligned} MC_C^i &= MC_C^{i-1} + MD_C^{i-1} r_{withdraw} - Expenditur e^{buy} + bx^{i-1} + a_0 \\ MD_C^i &= MD_C^{i-1}(1 - r_{withdraw}) + (1 - b)x^{i-1} - a_0 + Interest_C^{i-1} \\ MD_P^i &= MD_P^{i-1} + \{Sales_P^{i-1} + Interest_P^{i-1} - (Cost_P^{i-1} + \sum_{k \in \{C\}} wage_k^{i-1})\}(1 - cTaxrate) + Repayment_P^{i-1} \\ MC_G^i &= MC_G^{i-1} + Tax revenue^{i-1} - \sum_{k \in \{C\}} wage_k^{i-1} - Expenditur e^{poricy} \end{aligned} \quad (2)$$

where,

$$\begin{aligned} x^i &= wage^i(1 - iTaxrate) & , & \quad wage^i = Const \ wage^i + Bonus^i \\ Consumption \ budget^i &= a_0 + bx^{i-1} + MD^i r_{withdraw} & , & \quad Expenditur e^{poricy} = Market \ purchasing + Firm \ subsidy \\ Tax \ revenue^i &= \sum_{k \in \{C\}} (wage_k^{i-1} iTaxrate) + \sum_{k \in \{P\}} \{Sales_P^{i-1} + Interest_P^{i-1} - (Cost_P^{i-1} + \sum_{k \in \{C\}} wage_k^{i-1})\} cTaxrate \\ i &: \text{Fiscal period} & , & \quad \text{suffix: Agent type} \\ MC &: \text{Cash possessed by agent} & , & \quad MD: \text{Deposit of agent in the bank} \\ r_{withdraw} &: \text{Ratio of withdrawal money to total deposit} & , & \quad Interest: \text{Blance of interest paid by the bank} \\ Cost &: \text{Expenditure of retailer to buy rawmaterials} & , & \quad Expenditur e^{buy}: \text{Expenditure of consumer to buy retail product} \\ cTaxrate &: \text{corpration tax rate} & , & \quad iTaxrate: \text{Income tax rate.} \\ Repayment &: \text{Decrease agent in the balance of loaned money} \end{aligned}$$

A set of actions of each agent composes a unit of period and one period is assumed to correspond to one month in the real system. During each period, some of the state variables of agents will change in its value due to the interaction between agents. At

the end of each fiscal period, each agent settles the accounts with double-entry bookkeeping method. By summing up the calculated data of all agents, an input-output table of the artificial system is defined and GDP is obtained at every fiscal period. The rules of actions of each type of agent are assumed as shown below.

2.1 Consumer Agent

Consumer agents work at one of the producers, or at the government in case of public consumers, get wages, pay tax depending on their income and buy products supplied by retailers according to their utility functions within the limit of cash at hand. Cash at hand is defined as the sum of Keynesian consumption function and the money withdrawn from their bank accounts according to the withdrawal ratio as shown in equation (2). When there are goods of same class available in the market with different prices, the consumer is assumed to select and purchase the cheapest one among them. The utilities for each class of products are given randomly with uniform distribution at the start of the simulation.

2.2 Producer Agent

Producers hire consumers as employees, pay wages, make production plans, produce products of several types, supply and sell them in the market, pay tax depending on their profit and deposit and withdraw money in the bank at every fiscal period. The wages are composed of a fixed salary which is assigned to each consumer randomly between lower and upper limits and a bonus which is given when the profit of the producer is positive. The amount of bonus is defined as the bonus ratio times of the producer's surplus money and uniformly assigned to each of the employees.

2.2.1 Retailer and Wholesaler

The retailers and wholesalers decide both the amount and price of each class of products, where the price is increased or decreased depending on the amount of goods in stock at the end of the previous period. The amount of production is decided so that the probability of the occurrence of being out of stock is less than 5%, which is estimated based on the total sales during the most recent 10 periods. When the estimated price is less than the running cost per product, the minimum price is set to be the running cost. On the other hand, when the estimated amount of production is less than 70% of the production capacity, the minimum amount of production is set to be that amount. The production capacity Y is defined by a Cobb-Douglas type function as shown in equation (3), where K is the number of equipments for production, L is the number of employees and α is 0.25.

$$Y = AK^\alpha L^{1-\alpha} \quad (3)$$

Retailers and wholesalers have initially one unit of equipment and increase the number one by one during the simulation by buying equipment from equipment maker when the production at maximum capacity continues for more than critical times. All of the necessary amount of money for investment is financed by the bank

and it is constantly paid back every period for constant repayment periods. Additional investment during the repayment periods is assumed to be not allowed.

Retailers and wholesalers also have a bankruptcy rule. When the period of no sales with respect to a certain class of products continues for more than critical times, the producer quits production of that class of product. The producer goes bankrupt when he quits the production of all classes of products.

In addition, retailers require one unit of raw material for the production of one unit of product and buy the necessary amount of material in the market according to their production plans. When the amount of material available in the market is less than the required number, the amount of products to be produced is limited to that amount.

Retailers also have a layoff rule. When the period of deficit continues for more than a critical time, one of the employees is laid off and assigned to the producer whose amount of accumulated profit is the largest among producers.

2.2.2 Equipment Maker

The equipment maker produces equipment according to the requirement from the retailers and wholesalers within the limit of production capacity. The price of the equipment is assumed to be constant in the present study.

2.3 Bank Agent

The bank keeps surplus money of other agents in their bank accounts, and lends money as a long term loan to producers according to their demands for investment with the interest rate of 3%. The bank also pays wages to the employees and pays tax to the government according to the interest income. The bank also lends money as a short term loan to the producers according to their requirement when their working capital becomes short. In the present study, the initial amount of funds in the bank is set to be very large so that there is no limitation on lending money to meet the demand of producers except that additional requirement of long term loans is not fulfilled during the repayment stage.

2.4 Government Agent

The government collects tax from the producers and consumers in the form of corporation tax and income tax respectively, pays wages to the public employees and spends the resultant money for public spending according to the expenditure policy in every fiscal period. Corporation tax is only collected when the profit of the producer is positive and tax rate is assumed to be constant. Income tax is collected according to the consumer's income and tax rate is also assumed to be a constant value. The wage of public employees is determined at each fiscal period so that it equals the average value of private employees' wage as the sum of fixed wage and bonus.

As the expenditure policies, market purchasing, firm subsidy and the combination of them are tested, assuming the extreme cases of effectiveness in public spending.

The market purchasing is an expenditure policy where the government directly purchases goods in the market with the market price. When there are goods of the

same class available in the market with different prices, the government selects and purchases the cheapest one among them. This policy corresponds to the extreme case of efficient government spending where the government orders jobs to the firms in a completely competitive situation with the same price level expected in the market

The firm subsidy is an expenditure policy where the government evenly distributes funds to producers without any limitation of their use. This policy corresponds to the extreme case of inefficient spending where the government orders jobs to the firms with a much higher price level than expected in the market or pays money for the jobs which have no public meaning in the society.

3 Simulation Conditions

A simulation program has been constructed using Microsoft Visual C++ with object oriented programming, where agents are represented as objects programmed as instances of classes. The numbers of agents are 100, 20, 3, 1, 1, 1, for consumer, retailer, wholesaler, equipment maker, bank and government respectively. The influences of expenditure policies by the government, tax rate and consumption function of consumers on the macroeconomic behavior of the system such as GDP, average price, etc are analyzed and compared with that of the condition without a government. The distributions of various factors, including consumer's assets, producer's assets as well as total sales, maximum difference in the price of products at each fiscal period and the difference in GDP during each period are also analyzed to confirm the emergent behaviors of macroeconomic phenomena.

Table 1. Simulation conditions.

(a) Parameter values of the base run		(b) Initial conditions of which change influence during each run of simulation	
Maximum fiscal periods	360	Consumer deposit	30000~50000
Number of consumer	100	Capital of R and W	80000~160000
Number of retailer	20	Capital of equipment maker	200000~220000
Number of wholesaler	3	Capital of bank	96000000~104000000
Number of equipment maker	1	Price of wholesaler products	130~160
Number of bank	1	Price of retailer products	2850~3150
Fixed salary	7000~7500	A in equation (3) for W	300~200
Bonus ratio	0.95	A in equation (3) for R	18~8
Number of product class	12		
Withdrawal ratio	0~0.5	(c) Variable parameters as experimental levels	
Loan interest	3%		
Deposit interest rates	0.50%	Without government	With government
Repayment period	120	Number of government employees	0
Investment value	500000	Number of retailer employees	88
Critical flag number for investment	10	Number of wholesaler employees	9
Critical flag number for to stop production	20	Number of equipment maker employees	2
Critical flag number for dismissal	5	Number of bank employees	1
The lower limit of production	70% of its production capacity	Income tax rate	10~40%(1.0% intervals)
		Corporation tax rate	10~40%(1.0% intervals)
		Budget ratio of firm subsidy	0~1(0.1 intervals)
		Budget ratio market purchasing	0~1(0.1 intervals)

Simulation conditions are given in the Table 1, which are divided into three tables, fixed parameters whose values are constant, initial conditions whose values may change during each run of simulation, and simulation parameters which are constant but changed for each run of simulation to clarify their influence on macroeconomic behavior in the present artificial economic system. As shown in Table 1(a), each run

of simulation includes 360 periods and repayment period of producers is assumed to be 120. Expenditure policies of government and tax rate are changed as experimental conditions as shown in Table 1(c) and their influence on GDP or other economic factors are investigated.

Among these values, number of agents, bonus rate and initial capital of agents are determined so that they are almost minimal under the condition of the occurrence of stable fund circulation.

4 Simulation Results

4.1 Emergent Properties in the Present Artificial Economic Systems

Before analyzing the influence of public expenditure and tax rate on GDP, emergent behavior of macro-economic phenomena in the model such as the distributions of various factors are analyzed. Figure 1 shows the chronological change in GDP and Gini coefficient of consumers under the non-government condition. It is noted that GDP shows cyclic up-and-down movements in time, representing business cycles. The cycle of this long term business cycles is almost coincident with the repayment period 120. The primary cause of these business cycles is discussed in the authors' previous work [8] and considered as a result of finance and repayment of funds by producers for capital investment. The capital investment occurs as a result of emergent behavior due to the interaction among agents via market. Although initial assets of consumers, denoted by consumer's deposit in Table 1, is given by a uniform random number between 30000 and 50000, inequality of consumer's assets emerges and Gini coefficient varies between 0.2 and 0.6 during the simulation as shown in Figure1. It is also noted that the Gini coefficient shows peak values when GDP becomes local maximum although their behaviors are not so similar.

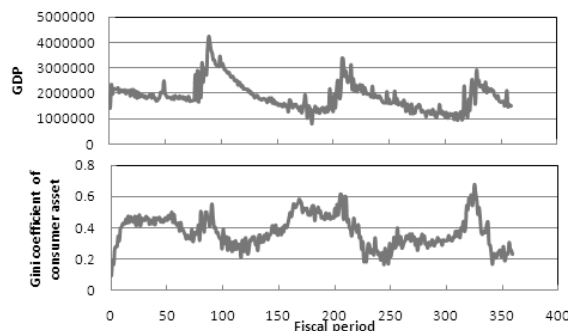


Fig. 1. Cyclic change on GDP(upper) and Gini coefficient (lower) under the condition without government.

Figure 2(a) shows the relationship between consumer's assets and the population who have that amount of assets, namely, the consumer's assets distribution. It is

clearly shown that consumer's assets distribution is represented by a power law distribution. It is noteworthy as shown in Figure 2(b) that the distribution of maximum difference in the price of same product class purchased in the market in each fiscal period is also represented by a power law distribution. Note that in the present model, the prices of the product of same product class in the market are in principle different, depending on the producers. Similarly, it is confirmed that the total sales and total assets of retailers also shows power law distribution.

As the power law distribution is one of the features of complex system [9], it is considered that these inequality behaviors of various factors such as consumer's assets, etc. are the properties emerged as a result of interactions among consumers and producers.

In addition, the absolute values of the change in GDP during each fiscal period also shows power law distribution as shown in Figure 3. Namely, although the long term business cycles are due to the finance and repayment of producers for capital investment, the fact that small short term variations shown in Figure 3 are represented by power law distribution suggests that the GDP in the present calculation is also an emergent property of artificial economic system as a result of interactions among agents.

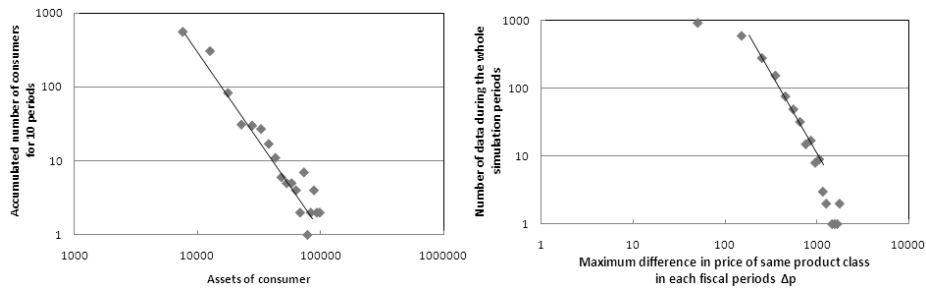


Fig. 2. (a) Consumers' asset distribution (left) and (b) distribution of the maximum difference in the market price of same product class (right).

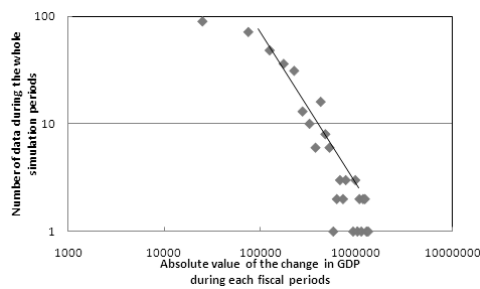


Fig. 3. Distribution of absolute value of the change in GDP during every fiscal period

4.2 Influence of Expenditure Policies of the Government on the Behavior of Macro-economy of the Artificial Economic System

Under the condition with constant tax rate, the influences of two types of expenditure policies of government on GDP are analyzed and calculated GDP level is compared with the GDP without government. Two types of expenditure policies are market purchasing and firm subsidy which are defined as the extreme cases of efficient and inefficient way of public spending, respectively.

Figure 4 shows chronological change in GDP and average price of products in the market where these factors under the expenditure policy of market purchasing and firm subsidy are compared with those in the case without government. It is noted in Figure 4 that during the whole periods, the level of GDP is larger in the case of market purchasing and smaller in the case of firm subsidy than that in the case without government. The average price of products shows similar behavior with that of GDP. It is also noted in Figure 4 that there is a tendency for the price to increase or decrease as the GDP is increasing or decreasing. Thus, inflation or deflation proceeds when GDP is increasing or decreasing according to the present model.

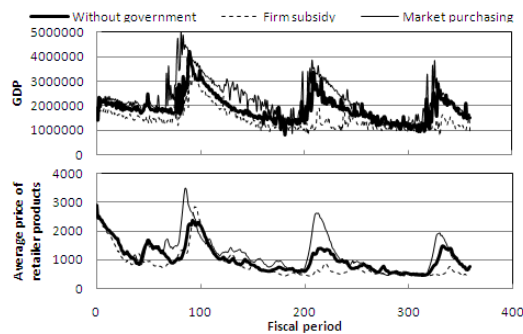


Fig. 4. Influence of expenditure policies of the government on GDP (upper) and average price of retailer products (lower).

4.3 Influence of Tax Rate on GDP

Influence of tax rate is dependent on public spending. Therefore, the ratio of market purchasing has been changed and the influence of income tax rate and corporation tax rate on GDP is analyzed for various ratios of market purchasing. Here, the ratio of market purchasing is defined by the amount of expenditure for market purchasing divided by the sum of the amount of expenditure for market purchasing and firm subsidy. Marginal propensity to consume is assumed to be 0.7 and GDP averaged for 360 periods is employed as the value of GDP.

The influence of tax rate on GDP is shown in Figure 5 for various ratios of market purchasing. In case of 100% of the ratio of market purchasing, GDP increases with increasing income tax rate as shown in Figure 5(a). This tendency is considered to

result because the money which is to be transferred to the consumer's bank account in the form of deposit is collected by the government and consumed for buying products in the market, thus increasing the demand in the market.

In case of 0% of market purchasing, that means 100% of firm subsidy, GDP remarkably decreases with an increase in income tax rate. This tendency is considered to result because government funds obtained by collecting tax are transferred to the firm's bank account almost without increasing consumer's income and money supply in the market. It is noteworthy that this negative correlation of GDP with income tax rate is observed in Figure 5(a) when the ratio of market purchasing is less than 90%. This result suggests that if a negative correlation of GDP with income tax rate is observed in the real system, it possibly means that the government expenditure includes more than 10% of inefficient way of public spending. The comparison of the present result with real data will be discussed in the next section.

The multipliers of the reduction of income tax rate on GDP are also calculated and it was revealed that the calculated multiplier varies between 0 and 1.3, corresponding to the ratio of market purchasing between 90% and 0%.

The influence of corporation tax is shown in Figure 5(b) for various ratios of market purchasing. In contrast to the influence of income tax rate, GDP increases with an increase in corporation tax rate regardless of the ratio of market purchasing. This tendency is considered to result because, under the condition of present study where overseas market and labor market are neglected, the money which is to be transferred to the firm's bank account is collected by the government and more or less consumed in the market, thus increasing the demand in the market. Even in case of 100% of firm subsidy, tendency is the same because the redistributed funds to the firms are partly transferred to the consumer, resulting the increase in the demand in the market.

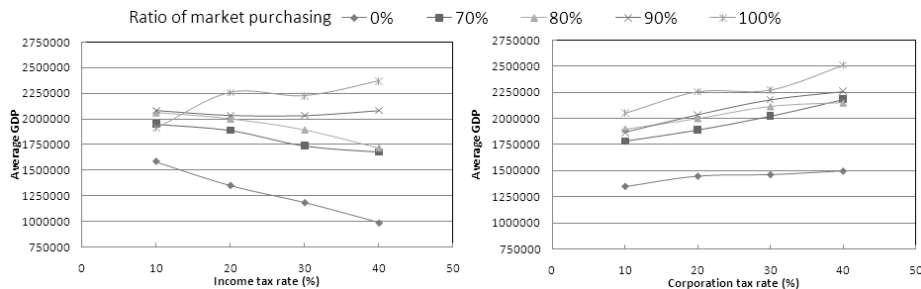


Fig. 5. (a) Influence of income tax rate (left) and (b) influence of corporation tax rate (right) on GDP averaged for 360 periods

4.4 Influence of Marginal Propensity to Consume on GDP

The influence of marginal propensity to consume on GDP is analyzed for various ratios of market purchasing, where income tax rate and corporation tax rate are both fixed to be 20%. It was confirmed as shown in Figure 6 that GDP increases with an increase in the ratio of market purchasing and decreases with a decrease in the marginal propensity to consume regardless of the ratio of market purchasing. GDP

level exceeds that of non-government condition when the ratio of market purchasing is more than 70% or 80%, depending on the marginal propensity to consume.

It was also revealed that the multipliers of the reduction of income tax rate on GDP decreases with a decrease in marginal propensity to consume.

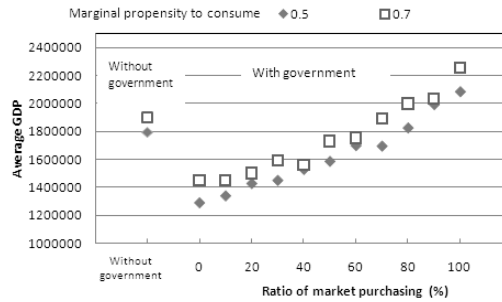


Fig. 6. Influence of marginal propensity to consume on GDP averaged for 360 periods.

5 Discussions

As shown in Figure 4, there is a tendency for the price to increase or decrease as the GDP increases or decreases. This tendency is compared with real data. The chronological data of GDP and consumer prices in G7 countries for more than 10 years are available in the IMF world economic outlook database [10]. Figure 7 shows the relationship between annual growth rates of GDP and increasing rates of consumer's prices in Japan during 1980 to 2010. As shown in Figure 7 that positive correlation between growth rate of GDP and increasing rate of consumer's prices is observed. It is noted that inflation or deflation is observed in most of the 23 years of positive GDP growth rate or in most of the 7 years of negative GDP growth rate. This result is qualitatively coincident with the calculated result of present study.

Influence of income tax rate and corporation tax rate on GDP calculated in the present study has been compared with the observed tendency in the real economic system where estimated results using "Short-Run Macroeconometric Model for Japanese economy" reported in the literature [11,12] are assumed to be the real data. Table 2 shows the estimated multipliers on GDP when income tax or corporation tax is reduced for 3 years by the amount corresponding to 1% of nominal GDP [12].

As shown in Table 2, reduction of the income tax rate will result in the increase of GDP in the real economy in Japan. The multipliers of tax reduction on GDP range between 0.2 and 1.2, which are close to the calculated value if government expenditure in the real system is assumed to include more than 10% of inefficient way of public spending.

From this point of view, the present results on the influence of income tax rate on GDP is in good agreement in its tendency with the real data, if it is assumed that the way of public spending by the government is not perfectly efficient, but includes some amount of inefficient factors. This might be reasonable to occur because, when

the government orders some job to the firms, firms may have a strong incentive to make contract with higher prices, while public employee has, generally speaking, comparatively weak incentive to decrease the prices.

As for the influence of corporation tax rate on GDP, on the other hand, there is a big difference between calculated and observed results. Namely, the reduction of corporation tax rate will also result in the increase in GDP in the real economy as shown in Table 2. This result is completely opposite in tendency with the calculated result as shown in Figure 5(b) where GDP decreases with decreasing corporation tax rate. This is considered because present model assumed to neglect labor market, international trading and advance into overseas market.

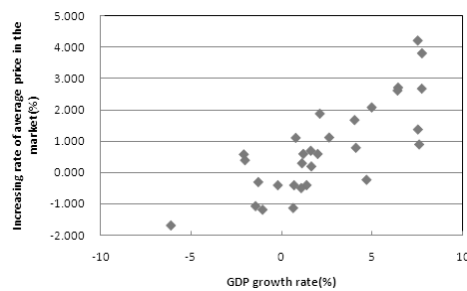


Fig. 7. Relationship between increasing rate of average price and GDP growth rate.

Table 2. Estimated multipliers on GDP when tax rate is reduced

Year	Effect of income tax reduction (1% of Nominal GDP)	Effect of corporation tax reduction (1% of Nominal GDP)
2005	0.24	0.45
2006	0.85	0.97
2007	1.17	1.10

6 Conclusion

An agent based model of an artificial economic system including government has been developed based on the authors' previous model. Using the model, the influences of public policies on GDP and related emergent behavior of macro-economic phenomena have been analyzed and the following results have been obtained.

1) Although the assets of agents are initially assigned by uniform random numbers, inequality of assets emerges during the simulation, and the distributions of various factors such as consumers' assets, maximum difference in market price in each fiscal period, total sales of retailers and the change in GDP during every period are represented by power law distribution. This suggests that various macro factors emerges due to the interaction between agents in the present model

2) As a result of analyzing the influence of expenditure policies on GDP, employing market purchasing and firm subsidy as the extreme cases of efficiency in public spending, it is revealed that market purchasing has a positive influence on the

GDP and firm subsidy has a negative influence. GDP increases with an increase in the ratio of market purchasing, and GDP level exceeds that of non-government condition when the ratio of market purchasing is more than 70% or 80%, depending on the marginal propensity to consume.

3) It is also found that GDP increases with a decrease in income tax when the ratio of market purchasing is less than 90%, while GDP increases with an increase in corporation tax rates, and the average price increases or decreases when GDP increases or decreases. These results have been compared with the real data in Japan and it is revealed that the obtained results of present study reproduce the feature of real economy observed in Japan except for the influence of corporation tax, if it is assumed that public spending includes more than 10% of inefficient factors. The calculated influence of corporation tax on GDP is not consistent with the real data, the reason of which is considered that present model neglects the labor market and international transactions.

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