Analysis of influential factors responsible for the effect of tax reduction on GDP

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Abstract. The factors responsible for the effect of a tax reduction on GDP are analyzed using both agent-based modeling (ABM), based on the authors' previous study, and a derived set of equations for tax reduction multipliers, based on Morishima's economic linkage table. The findings show that, under the balanced finance condition of government, the tax reduction multiplier is determined by the difference between the increase in demand by consumers or firms as a result of the tax reduction and the decrease in demand by the government. To increase the effect of a tax reduction, it is necessary that the increased disposable income of consumers or firms, as a result of the tax reduction, is more likely to be used for consumption and investment.

Keywords. Agent-based modeling, Tax reduction, Multiplier, Public expenditure, Balanced finance, Executive compensation, Investment.

1 Introduction

Agent-based modeling (ABM) is widely used in social simulations to explain or understand social phenomena [1]. One important research field is the application of ABM to macroeconomic systems, although these systems are very complex and include various kinds of agents and their interactions. When using ABM, it is considered important that the model be as simple as possible, based on "KISS Principle," to understand the most essential mechanisms of the phenomena in question [1]. However, it is also important to consider all factors required to reproduce the desired phenomena, because the structure of the artificial system should be the same as that of the real system to enable the characteristics to emerge as they do in the real system. Establishing the factors essential to reproducing the desired characteristics of the system can be done using a series of computer experiments in which only one constituent factor of the model is changed at a time, while the other factors are held constant [2, 3].

A number of ABM-based research studies have focused on various macroeconomic aspects, such as business cycles, innovation, economic growth, the role of banks, monetary policies, industrial dynamics, and wealth inequality [4-7]. Most of these

studies reported some new finding, but the model constitutions were different in each case. This makes it difficult to identify the crucial assumptions of each model and to what extent the assumptions are important in reproducing the phenomena under concern. Researchers have also developed relatively more practical models that simulate multiple-market economic structures as elaborately as possible [8, 9]. However, given the nature of these economic phenomena, these studies have not fully clarified the structural factors of the model that are important for their reproducibility.

Another potential area for applying ABM in the real world is government policy formulation, in areas such as tax reduction and public expenditure. According to Keynes' multiplier theory [10], government public expenditure and tax reductions are effective policies to promote a macro economy. However, the multipliers of these public policies are relatively small when compared to the values expected by the marginal propensity to consume, the reason for which is not well understood.

Motivated by this lack of understanding, the authors constructed a simple, artificial economic model consisting of consumers, three types of producers, a bank, and a government. The conditions required for the model to reproduce the positive influence of a tax reduction on GDP were then analyzed, with the intention of revealing and explaining the mechanism that makes the public policy multiplier so low. The findings show that inefficiency in government expenditure, executive compensation, and internal funds for investment are all factors responsible for the positive influence of a tax reduction on GDP. Here, inefficiency of public expenditure is defined as the ratio of firm subsidies to the sum of firm subsidies and market purchases [11, 12].

In the present study, additional simulations were conducted to clarify the mechanism explaining why the above factors in the model are responsible for reproducing the positive influence of a tax reduction on GDP. In addition, we derived a set of equations for the tax reduction multiplier based on our revised version of Morishima's economic linkage table [13], and compared the influence of the above-mentioned factors against the results calculated using ABM.

In general, theoretical approaches in macroeconomics that assume complete equilibrium between demand and supply and neglect the diversity of agents cannot describe complex systems well. However, these approaches have the advantage of being able to describe the mechanism behind the relationships among the influential factors. In explaining the influence of a tax reduction on GDP, important factor would not be the price in equilibrium, but the flow of funds among agents. In such cases, confirming whether the results calculated using ABM can be explained by theoretically derived equations could be an effective measure to validate the ABM study.

2 Simulation Model

2.1 Outline of model

The agent-based model of the artificial economic system in the present study comprises consumers, producers, a bank, and a government as autonomous decision-making agents. Consumers and producers are each divided into three types of agent.

Each agent is heterogeneous in its state variables and other parameters included in their action rules. Table 1 outlines the agents and their action rules.

Table 1. Outline of agents and their action rules.

Agent	Type	Output to be supplied	Product type to purchase	Outline of action rules
Consumer				Consumers work and obtain wage at producer, bank or government , pay tax, and purchase consumption goods. Income is divided into the money for consumption and deposit by Keynesian consumption function. The consumer is assumed to select and purchase the cheapest one among products.
	Worker	The labor force for firms	Consumption goods	
	Executive	Management for firms		
	Public workers	The labor force for government		
Producer				Producers hire consumers, produce and sell products, pay wages and tax.
	Retailer	Consumption goods	Consumption goods, Materials, Equipment	The retailers and raw material makers decide both the amount and price of each class of product according to its state of amount of stock.
	Raw-material-maker	Material goods	Consumption goods, Equipment	
	Equipment-maker	Equipment	–	The equipment-maker produces equipment in line with the requirements of the retailers and wholesalers.
Bank	Bank	The fund for producers	–	The bank keeps the surplus money of other agents in their respective bank accounts, and lends money.
Government	Government	Redistribution of wealth	Consumption goods	The government collects tax from other agent, pays wage to public worker and spends remaining money on expenditure of public policy.

2.2 Sequence of actions

The set of actions for each agent comprise period-based units, where one period is assumed to correspond to one month in the real system. During each period, agents act and interact with each other according to a sequence of seven steps. At the end of the sequence for each period, a GDP value is calculated based on an input-output table obtained by summing each agent's account data. The details of the seven steps describing the sequence of agents' actions are as follows:

1. Agents pay any unpaid tax for the previous period. After paying tax, agents create a budget plan for consumption, paying wages, or public spending.
2. Raw material makers decide on the amount and price of products to be produced, produce several types of raw materials, and supply these to the material goods market.
3. Retailers decide on the amount and price of products to be produced, purchase raw materials in the material goods market, produce several types of consumption goods, and supply these products to the consumption goods market.
4. Consumers, retailers, raw material makers, and the government purchase products in the consumption goods market.
5. Retailers and raw material makers judge the necessity of investment based on total sales in previous periods and, if necessary, invest in equipment.
6. Each firm pays wages to employees and executive compensation to the executive, and the government pays wages to public workers.
7. Each agent settles its accounts using the double-entry bookkeeping method. They calculate their income and profit for the current term, and then determine the amount of tax to be paid based on these figures. If necessary, a retailer may dismiss a worker, depending on the profits of the current and previous periods, or may decide to stop producing a certain type of product altogether, depending on its total sales.

2.3 Outline of agent's decision-making rules

2.3.1 Consumer agent behavior rules

Consumers create a budget for consumption, E_b^t. This budget is defined as the sum of the terms based on after-tax income I^t (1-r_{i_tax})(represented by the Keynesian consumption function [10]), and a withdrawal ratio of r_{wd} times their bank deposit, D^t, at each fiscal period, t. The formula for the budget is shown in Equation (1). Here, r_{i_tax} is the income tax rate, a is the consumer's basic consumption, and b is the marginal propensity to consume as per the Keynesian consumption function. The withdrawal ratio, r_{wd}, is selected randomly for each agent during each period.

$$E_b^t = a + bI^t(1 - r_{i\_tax}) + r_{wd}^t D^t \tag{1}$$

When purchasing products in the consumption market, consumers select and purchase products within the limit of their consumption budget according to the utility function for each class of products given by Equation (2). Here, w_i is the weight of the utility for each type of class, i, which is randomly assigned to each agent, x_i is the number of products purchased, p_i is the price of a product, andα is an exponent of x_i ranging between 0 and 1. When there are goods of the same class available in the market at different prices, consumers select and purchase the cheapest available.

$$\max u = \sum_i w_i x_i^{\alpha} \qquad s.t. \sum_i p_i^t x_i \le E_b^t \tag{2}$$

2.3.2 Producer agent behavior rules

The retailers and raw material makers both decide on the amount and price of the products they will produce. The price of a product in a product class increases or decreases, depending on how much of the product is in stock at the end of previous period. Production levels are decided in such a way that the probability of being out of stock is less than 5%. This is estimated based on the total sales over the most recent ten periods. If the estimated production is less than 70% of production capacity, the minimum amount of production is set at the 70% level.

The production capacity, Y, is defined by the Cobb-Douglas production function [10], as shown in Equation (3), where K is the number of units of equipment used in production, L is the number of employees, and α is assumed to be 0.25. In addition, A is a proportionality constant assigned randomly to each producer between a lower and upper limit. It is assumed that this value is peculiar to each producer, and represents that producer's technical capability.

$$Y_i(K, L) = A_i K^{\alpha} L^{1-\alpha} \tag{3}$$

Retailers and raw material makers initially have one unit of equipment and a specified number of employees. They will invest to increase their production capacity when production at maximum capacity continues beyond a critical period, based on the expected profit. They decide to invest when the expected financial benefit given

by Equation (4) is positive. Here, p_i is price of a product of class i, c is the variable cost per unit product, r_0 is the borrowing interest rate, F is the amount borrowed to buy one unit of equipment, N is the repayment period, and w is the fixed wage per employee. It is assumed that the depreciation period of the equipment is the same as the repayment period.

$$\Delta\pi_K = \max_i[(p_i^t - c_i^t)\{Y_i(K+1,L) - Y_i(K,L)\} - (r_0 + 1/N)F] \tag{4}$$

When they decide to invest, half of the necessary funds are financed by the bank and the rest is financed by the firm's internal funds. The funds for investment financed from the bank are repaid as a fixed amount in each period and for a constant number of consecutive repayment periods. During the repayment periods, additional investment is no longer allowed when the total number of investments exceeds a certain upper limit.

The equipment maker produces equipment in accordance with the requirements of the retailers and raw-material-makers within its production capacity limit. In the present study, the price of the equipment is assumed to be constant.

One executive and several workers are assigned to each of the producer agents. The producers pay a wage to workers and the wage plus executive compensation to the executive in each period. The executive compensation in the real world comprises a paid salary, a bonus, and long-term incentives. In the present model, the paid salary is assumed to be the same as the wages paid to workers, the long-term incentives are ignored, and only the bonus is defined as the executive compensation which is paid from after-tax profits. Wages comprise a fixed salary, randomly assigned to each employee between a lower and an upper limit, and a bonus given when the producer's profit is positive. The total spent on wages for each producer, E^t_w, is given in Equation (5), where W_f is the total fixed salary, W^t_b is total amount paid in bonuses, EC^t is the amount paid as executive compensation, π^t is the profit before tax, and AC is accumulated earnings. In addition, W^t_b is defined as $r_b\pi^t$, where r_b is the bonus ratio, and EC^t is the executive compensation ratio multiplied by the after tax profit, and defined as $\pi^t\ (1\text{-}r_b)(1\text{-}r_{c\_tax})r_e$, where r_{c_tax} is the corporation tax rate and r_e is the executive compensation ratio.

$$E^t_w = \begin{cases} W_f & \text{if} \quad \pi^t < 0 \\ W_f + W^t_b & \text{if} \quad \pi^t > 0 \text{ and } AC < 0 \\ W_f + W^t_b + EC^t & \text{if} \quad \pi^t > 0 \text{ and } AC > 0 \end{cases} \tag{5}$$

Retailers also have a dismissal rule and bankruptcy rule, details of which are presented in the ODD protocol given in the Appendix.

2.3.3 Bank agent behavior rules

The bank retains the surplus money of other agents in their respective bank accounts, earns interest on long-term and short-term loans, and pays wages to its employees and taxes to the government in line with its interest income. The bank lends money as long-term loans to producers in line with their demands for investment, charging a 3%

interest rate. The bank also lends money as short-term loans to producers to meet their requirements when their working capital to pay fixed wages and/or purchase raw material becomes sufficiently small. In the present study, the initial funds available to the bank is set to be very large so that there is no limitation on lending to producers, except in the case when long-term loans are not fulfilled during the repayment periods.

2.3.4 Government agent behavior rules

The government collects corporation tax and income tax, pays wages to public employees, and uses the resultant money for public expenditure, according to their expenditure policy. Corporation tax is only collected when the profit of the producer is positive, and the tax rate is assumed to be constant. Income tax is also assumed to be constant and is collected according to the consumer's income. The wages of public employees are determined at each fiscal period so that they are equal to the average value of a private employee's combined fixed wage and bonus.

With regards to expenditure policies, market purchasing, firm subsidy, and combinations of the two are tested. Market purchasing is an extreme case of efficient public expenditure in which the government directly purchases goods at the market price. This policy corresponds to the case where the government places job orders with firms in a completely competitive situation at the market price. Firm subsidy is an extreme case of inefficient public expenditure in which the government distributes funds to producers evenly, without any limitations on their use. This policy corresponds to the case in which the government places job orders at a much higher price than expected in the market or pays money for jobs that have no economic value.

3 Simulation Condition

A simulation program was constructed using C++ with an object-oriented method. The simulation conditions are given in the three sub-tables of Table 2. Table 2(a) shows the fixed parameters with values that remain constant during the simulation. Table 2(b) shows the initial conditions. Here, the values are initially given by a uniform random number, but may change during each simulation run. Table 2(c) shows the simulation parameters as experimental levels. These are constant, but change in each simulation run to clarify their influence on macroeconomic behavior in the artificial economic system. In addition, the parameters displayed as a range of two values in Table 2 are assigned a uniform random number within this range when the simulation starts, or during the simulation.

The simulation conditions as experimental levels are divided into two categories: an analysis of the income tax rate and an analysis of the corporation tax rate. In the former category, the withdrawal ratio of deposits was changed during the experiment to change the marginal propensity to consume. In the latter category, the executive compensation ratio was changed to change the level of the use of internal funds. In addition, in the case of latter category, the number of investments during the overall simulation period were also analyzed as a function of the corporate tax rate.

Table 2. Simulation condition

(a) Parameter values of the base run

Maximum fiscal periods t	360
Number of consumers	150
Number of retailers	30
Number of raw material makers	4
Number of equipment makers	1
Number of banks	1
Deposit interest rates r_{int}	0.50%
Loan Interest r_θ	3%
Bonus ratio r_b	0.75
Number of product classes i	12
Weight of utility w	0.3～01.1
Basic consumption a	3000
Marginal propensity to consume b	0.7
Fixed salary	7000～7500
Repayment period N	120
Investment value F	500000
Critical flag number to quit production	20
Critical flag number for dismissal	5
Lower limit of production	70% of its capacity

(b) Initial conditions whose value may change during each run of simulation

Consumer deposit	30000～50000
Capital of retailer and raw material maker	80000～160000
Capital of equipment maker	200000～220000
Capital of bank	96000000～104000000
Prices of raw material makers' products	130～160
Prices of retailers products	2850～3150
A in equation (7) for raw material maker	200～300
A in equation (7) for retailer	8～18

(c) Simulation conditions as experimental levels

	Analysis of income tax rate			Analysis of corporation tax rate		
	Basic	Consumers' cargnal propensity to deposit		Basic	Producers' marginal propensity to deposit	
		High	Low		Middle	Low
Income tax rate	10～30%(5% intervals)			20%		
Corporation tax rate	20%			10～30%(5% intervals)		
Budget ratio of firm subsidy	0～100%(10% intervals)			0～100%(10% intervals)		
Budget ratio market purchasing	0～100%(10% intervals)			0～100%(10% intervals)		
Withdrawal ratio r_{wd}	0～0.5	0～0.2	0～0.8	0～0.5		
Executive compensation ratio r_e	95%			95%	75%	65%

4 Simulation Results

The influence of executive compensation and the financing rule using internal funds on the relationship between GDP and the corporation tax rate is shown in Fig. 1 as a typical result obtained in the authors' previous study. It was also found that inefficiency of public expenditure is another influential factor [12]. To understand the mechanism of how these factors affect the relationship between GDP and tax rate, additional simulations were conducted in the present study.

The market purchasing ratio represents the efficiency of public expenditure and the consumers' withdrawal ratio is an indicator of their marginal propensity to consume. Fig. 2 shows the dependency of the ratio of an increase in GDP due to an income tax reduction on the market purchasing ratio and the consumers' withdrawal ratio. Here, the ratio of the GDP increase was defined as the ratio of an incremental increase in the original value when the income tax rate was reduced from 30% to 10%. In Fig. 2, each plot represented by a square shows the interval between the first and third quartiles of the ratio of GDP increase, which was obtained by changing the initialization of the pseudo-random number generator in C++ in ten patterns. It is noted that, in Fig. 2, the ratio of GDP increase due to an income tax reduction becomes larger as the withdrawal ratio increases and the market purchasing ratio decreases.

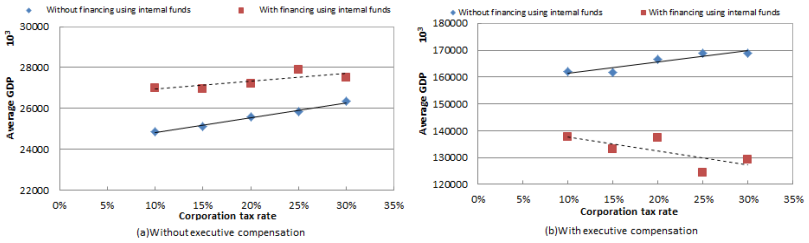

Fig. 1. Influence of executive compensation on the relationship between GDP and corporation tax rate: (a) without the financing rule using internal funds; and (b) with the financing rule using internal funds [12].

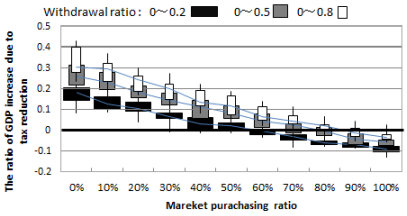

Fig. 2. Influence of consumers' marginal propensity to consume on the rate of change of GDP by income tax reduction.

Fig. 3 shows the dependency of the ratio of a GDP increase due to a corporation tax reduction on the efficiency of public expenditure for various executive compensation ratios. As shown in Fig. 3, the ratio of the GDP increase due to a corporation tax reduction increases as the executive compensation ratio increases and as the market purchasing ratio decreases (i.e., less efficient public expenditure).

To understand why the internal funds rule affects GDP, the number of investments was investigated as a function of the corporation tax rate for various levels of market purchasing ratios, as shown in Fig. 3. In Fig. 3(b), the tax reduction promotes investments. The reason for this tendency is that the increase in internal funds due to the tax reduction increases investment, because the decision to invest depends on the amount of internal funds in case of internal funds rule.

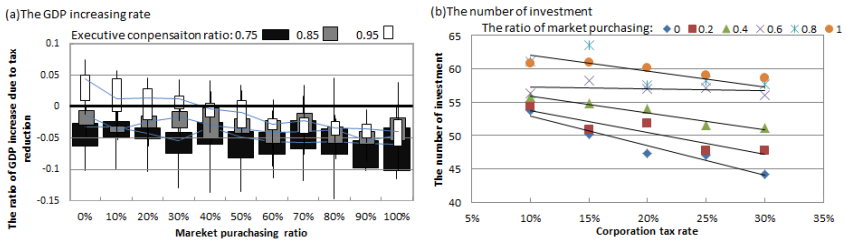

Fig. 3. (a) The influence of producers' marginal propensity to consume on the rate of change of GDP caused by a corporation tax reduction; (b) The influence of the tax rate and the ratio of market purchasing on the number of investments when executive compensation is 0.95.

5 Discussion

As explained in the introduction, the authors' previous work showed that inefficiency in government expenditure, executive compensation, and internal funds for investment are all responsible for the positive influence of a tax reduction on GDP. This was confirmed by the additional analysis in the present study described above. To understand why these factors affect the tax reduction multiplier, this section derives a set of equations for the multiplier based on our revised version of Morishima's economic linkage table [13].

Table 3. Revised table of economic linkages

		(1) Consumption goods Industry	(2) Capital goods Industry	(3) Workers w	(4) Executive e	(5) Rentiers r	(6) Management sector m	(7) Investment sector i	(8) Foreign trade f	(9) Government g	(10) Bank b	(11) Central bank c	(12) Foreign exchange stabilization fund s
1	Consumption goods	$-p_1X_1$		$p_1D_1^w$	$p_1D_1^e$	$p_1D_1^r$			p_1E_1	$(1-\eta)G_1$			
2	Capital goods		$-p_2X_2$					p_2I_2	p_2E_2	$(1-\eta)G_2$			
3	Wages	$wa_{31}X_1$	$wa_{32}X_2$	$-W$						wN^g	wN^b		
4	Depreciation	$p_1a_{41}X_1$	$p_2a_{42}X_2$					$-H$					
5	Imports	$\eta p_1^*a_{51}X_1$	$\eta p_2^*a_{52}X_2$						$-\eta p_1^*F$				
6	Taxes	$t_1(1+m)c_1X_1$	$t_2(1+m)c_2X_2$	$t_w W$	$t_e\alpha(1-t_m)\Pi$	$t_r(\overline{A}^r+r\overline{B}^r)$	$t_m\Pi$			$-T$			
7	Profits	mc_1X_1	mc_2X_2		$-\alpha(1-t_m)\Pi$		$\alpha(1-t_m)\Pi+\gamma-\Pi$	$-\gamma$		$\eta(G_1+G_2)$			
8	Interest					$-\overline{A}^r$		$-\overline{B}^i$	$-\overline{B}^f$	$-\overline{B}^g$	$-\overline{A}^b$	$-\overline{A}^c$	
9	Bond					$p_1\delta B^r$		$p_1\delta B^i$	$p_1\delta B^f$	$p_1\delta B^g$	$p_1\delta B^b$	$p_1\delta B^c$	
10	Time deposit					δQ^r					δQ^b	δQ^c	
11	Foreign exchange					$r(p_1^*\delta B_r^* - B_r^*)$			$r(D_1^* - S_1^*)$				rD_1^*
12	Money			$L^w-\overline{M}^w$	$L^e-\overline{M}^e$	$L^r-\overline{M}^r$	$L^m-\overline{M}^m$	$L^i-\overline{M}^i$			$L^b-\overline{M}^b$	$-\delta M^c$	$-\delta M^s$

In Morishima's table of economic linkages, the individuals are divided into workers, entrepreneurs, and rentiers, and firms and other sectors are divided into an investment sector, foreign trade, government, bank, and the rest. In addition, industrial outputs are assumed to be consumption goods and capital goods, the prices of which are assumed to be constant.

Some corrections need to be made for the purpose of analysis. To analyze the influence of a tax reduction on GDP, we divide the group of entrepreneurs in Morishima's table into executives, who pay income tax, and management sectors, who pay corporation tax. In addition, to consider the inefficiency of public expenditure, we divide government expenditure ***pG***, in Morishima's table into **(1-*η*)*pG*** for buying goods and ***ηpG*** for a subsidy, which is added to profit of firms, where ***η*** is the inefficiency of public expenditure. Table 3 shows the revised table of economic linkages,

5.1 Multiplier of income tax reduction

An income tax reduction increases the disposable income of workers and executives, thereby increasing the demand for consumption goods as a direct result. This increases the total sales of the consumption goods industry, which in turn increases the disposable incomes of workers and executives. Thus, the overall demand for consumption goods is given by Equation (6), if we ignore the demand of other sectors such as rentiers, where c_{we} is average marginal propensity to consume, ΔG is total value of the tax reduction, and $\eta$is the inefficiency of public expenditure.

$$
\begin{aligned}
& p_1\Delta X_1 = b_1 p_1\Delta X_1 + (c_{we} - b_3)(\Delta G) \\
& \quad where,\ b_1 = c_w(1-t_w)\,wa_{31}/p_1 + c_e(1-t_e)(1-t_m)\,\alpha\, mc_1/p_1 \\
& \qquad b_3 = c_e(1-t_e)(1-t_m)\alpha\eta + (1-\eta) \\
& \qquad c_{we} = \{c_w(-\Delta t_w W) + c_e(-\Delta t_e(1-t_m)\alpha\Pi)\}/\Delta G \\
& \qquad \Delta G = (-\Delta t_w W) + (-\Delta t_e(1-t_m)\alpha\Pi)
\end{aligned}
\tag{6}
$$

The GDP is given by Equation (7). Substituting Equation (6) into Equation (7), we can obtain the multiplier of the income tax reduction, as given in Equation (8).

$$\begin{aligned}\Delta Y &= p_1\Delta X_1 + p_2\Delta X_2 - \mu_1 p_1\Delta X_1 - \mu_2 p_2\Delta X_2 \\ &= (1-\mu_1)p_1\Delta X_1 + (1-\mu_2)p_2\Delta X_2 \\ &\quad \text{where, } \mu_1 = r(p_5^*/p_1)a_{51} : \textit{Ratio of import goods in the price of consumable goods} \\ &\quad \mu_2 = r(p_5^*/p_2)a_{52} : \textit{Ratio of import goods in the price of durable goods}\end{aligned} \tag{7}$$

$$\frac{\Delta Y}{\Delta G} = (1-\mu_1)\frac{1}{1-b_1'}(c_{we} - b_3) \tag{8}$$

In Equation (8), $\boldsymbol{b_3}$ shows the government's marginal propensity to consume, because it indicates the ratio of the government's consumption. Therefore, Equation (8) indicates that the multiplier of income tax reduction is determined by the difference between the average marginal propensity to consume of consumers and that of the government.

Assuming $\boldsymbol{c_w = c_e}$ and that substituting typical values in our simulation, as presented in the previous section, as $\boldsymbol{t_w = t_e = 0.2}$, $\boldsymbol{wa_{31}/p_1 = wa_{32}/p_2 = 0.6}$, $\boldsymbol{m = 0.25}$, $\boldsymbol{c_1/p_1 = c_2/p_2 = 0.8}$, $\boldsymbol{t_1 = t_2 = 0}$ into Equation (8), the multiplier of the income tax reduction is represented as a function of the efficiency of public expenditure and of the marginal propensity to consume, as shown in Fig. 4. It is noted that this tendency coincides with the relationship shown in Fig. 2, obtained from the simulation.

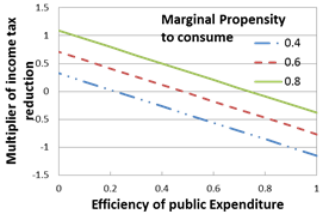

Fig. 4. The influence of efficiency of public expenditure and marginal propensity to consume on the multiplier of income tax reduction.

5.2 Multiplier of corporation tax reduction

The multiplier of the corporation tax reduction would be very small, if the internal funds increased as a result of the tax reduction is assumed not to be used for investment. Here, we derive the equation for the multiplier based on Table 3, assuming that β times the amount of the tax reduction is used for investment. This investment increases the demand for capital goods, which increases the total sales of firms. This increases the disposable income of workers and executives, and therefore, increases the demand for consumption goods. On the other hand, the government decreases public expenditure to compensate for the decrease in tax revenue under the balanced budget condition. The overall demand for consumption goods and capital goods are represented by Equations (9) and (10), respectively. Substituting Equations (9) and (10) into Equation (8), we obtain Equation (11), which represents the multiplier for the corporation tax reduction.

$$p_1\Delta X_1 = b_1 p_1\Delta X_1 + b_2' p_2\Delta X_2 + (b_4 - b_3)\Delta G$$
$$\text{where, } b_1 = c_w(1-t_w)a_{31}w/p_1 + c_e(1-t_e)(1-t_m)\alpha m c_1/p_1 \quad (9)$$
$$b_2' = c_w(1-t_w)a_{32}w/p_1 + c_e(1-t_e)(1-t_m)\alpha m c_2/p_2$$
$$b_4 = c_e(1-t_e)\alpha, \quad \alpha : \text{Ratio of executive compensation}$$

$$p_2\Delta X_2 = p_2 I_2 = \beta\Delta G \quad (10)$$
$$\text{where, } \beta : \text{Ratio of consumption for investment to the amount of tax reduction}$$

$$\frac{\Delta Y}{\Delta G} = (1-\mu_1)(b_2'\beta + b_4 - b_3)/(1-b_1) + (1-\mu_2)\beta$$
$$= A\beta + (b_4 - b_3)(1-\mu_1)/(1-b_1) \quad (11)$$
$$\text{where, } A : \text{Multiplier of firm's investment}$$

In Equation (11), the terms $\boldsymbol{\beta}$ and $\boldsymbol{b_4}$ represent the ratio of consumption for investment and executives' marginal propensity to consume, respectively, and their sum represents the ratio of consumption to the total amount of the tax reduction. Therefore, Equation (11) indicates that the multiplier of the corporation tax reduction is determined by the difference between the substantial marginal propensity to consume of firms and that of the government. Assuming $\boldsymbol{c_w = c_e = 0.9}$, $\boldsymbol{t_w = t_e = 0.2}$, $\boldsymbol{wa_{31}/p_1 = wa_{32}/p_2 = 0.6}$, $\boldsymbol{m = 0.25}$, $\boldsymbol{c_1/p_1 = c_2/p_2 = 0.8}$, $\boldsymbol{t_1 = t_2 = 0}$, and substituting these values into Equation (9) as typical values in the simulation, the multiplier of the corporation tax reduction is represented in Fig. 5. It is noted that the multiplier increases as the ratio of executive compensation increases, and as the efficiency of public expenditure decreases. This tendency agrees with that shown in Fig. 3(a). It is also noted that the multiplier increases as the value of $\boldsymbol{\beta}$ increases. This tendency agrees with the relation shown in Fig. 3(b). The reason why the utilization of internal funds for investment is an indispensable condition for reproducing a positive influence of a tax reduction on GDP in our simulation [12] is that firms must use part of their increased internal funds, as a result of the corporation tax reduction, for investment.

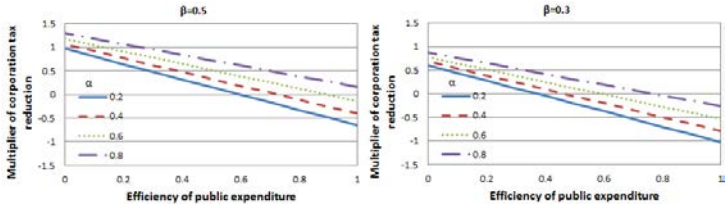

Fig. 5. The influence of the efficiency of public expenditure, α and β, on the multiplier of the corporation tax reduction. (α:executive compensation ratio, β:consumption ratio for investment)

6 Conclusion

The factors responsible for the effect of a tax reduction on GDP were analyzed using both ABM, based on the authors' previous study, and a theoretical derivation of a set of equations for the tax reduction multipliers, based on our revised version of Morishima's economic linkage table. As a result, the following findings were obtained.

1) In the ABM analysis, the ratio of the GDP increase due to the tax reduction increases as the efficiency of public expenditure decreases in the case of both income tax and corporation tax reductions. The ratio of the GDP increase also depends on the marginal propensity to consume, in the case of the income tax reduction, and on the executive compensation ratio and the frequency of investments in the case of the corporation tax reduction.
2) The equations for the tax reduction multiplier, which were derived in the present study, showed that the multipliers have the same dependency on the influential factors observed in ABM analysis. It was found that the most important factor in the case of the corporation tax reduction is the ratio of the increased consumption for investment to the total amount of the tax reduction.
3) The derived equations suggest that, under the balanced finance condition, the tax reduction multiplier is determined by the difference between the increased consumption of goods and the amount of investment by consumers or firms, and the decreased demand by the government. The critical amount of consumption for a positive multiplier becomes larger if public expenditure becomes more efficient.

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Appendix: Overview, Design concepts, Details Protocol

This appendix describes the model in terms of the Overview, Design concepts, and Details(ODD) protocol by Grim, et al.(2006).
Grim, et al. (2006))

1. Purpose

The purpose of this model is to experimentally elucidate the underline mechanism of the complex macroeconomic phenomena. The model also aim to clarify the conditions under which the model structures reproduce these phenomena in an agent-based artificial economic system where macroeconomic indicators emerge as a result of agents' actions and interactions. In the present study, the purpose of this model focuses on elucidating the model structure to reproduce the positive influence of corporate tax reduction on GDP and to obtain a clearer understanding of the mechanism behind this effect.

2. Entities, state variables, and scales

The entities included in this model are agents, goods, and markets, which are the minimum requirements of a macro economy of a nation. Agents include the following: consumers, comprising workers and executives of private sector firms and public workers; producers, comprising retailers, raw material makers, and an equipment maker; a bank; and a government. Goods include consumption goods for any agents, raw material goods for retailers, and equipment as capital goods for retailers and raw material makers. The market is divided into a consumption goods market and a raw material goods market. We assume that capital goods transactions take place directly between equipment makers and buyers. Fig. 1 shows the relationship between these entities, including the flows of goods, labor, and funds among the agents.

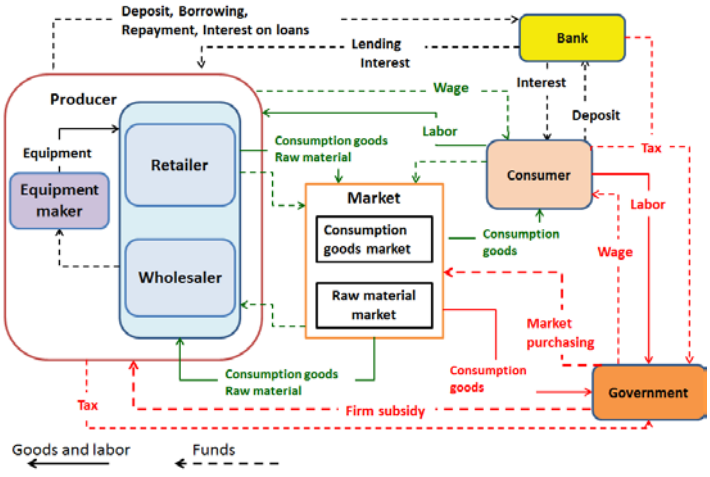

Fig.1 Outline of the entities and their relationships

The entities included in this model and their characteristics are described in Table 1.

Table.1 Entities included in the model and their characteristics

Entities	Descriptions
Agent	Characterized by agent id, agent type, cash it holds, deposit it holds and common behavioral rules as a buyer, a seller and an enterprise.
Consumer	Works for one of other agents, get wages, pay income tax and purchase consumption goods.
Retailer Producer	Produces consumption goods, ships them to the market for selling, pays wages and pays corporation tax. It also buys consumption goods.
Raw material maker	Produces raw material goods, ships them to the market for selling, pays wages and pays corporation tax. It also buys consumption goods.
Equipment maker	Produces equipment based on theorders from retailers and wholsellers, pays wages and pays corporation tax.
Government	Collects tax from consumers and producers, purchases consmption goods and supply subsidies to the firms as public expenditures.
Bank	Keeps deposits from consumers and producers, finance funds for investments of producers, pays or earns interest on deposits or on loans.
Goods	Characterized by 12 types of product classes, price, seller's id and buyer's id. There are no behaivoral rules for goods.
Consumption goods	Produced and supplied to the market by retailers, bought by sevral types of buyers.
Raw material goods	Produced and supplied to the market by raw material makers, bought only by retailers One-to-one correspondence is assumed between raw material goods and consumption goods, representing the smallest supply chain.
Equipment	Produced by an equipment maker according to the requirement for investments from retailers and raw materials makers.
Market	Characterized by a set of goods objects. Goods remained unsold at a time period are recognized by the corresponding seller agents at the beginning of next period as goods in stock.
Consumption goods market	Consumption goods are supplied by retailers and bought by buyers includling consumers, retailers, raw material makers and a government. An event of buying one of the goods is synchronously recognized by a corresponding seller agent, resulting in the payment by a buyer .
Raw material goods market	Raw material goods are supplied by raw material makers and bought by only retailers. Transactions are conducted with the same algorithm as the case of consumption goods.

State variables are divided into those for agents and those for other entities. Each agent belongs to a different category, according to its behavior, such as agents in general, buyers, enterprises, and producers. State variables for agents are divided into those for an agent's behavioral category and those peculiar to each type of agent. State variables for each behavioral category are described in Table 2, and the state variables peculiar to each type of agent are described in Table 3. The state variables of other types of entities are described in Table 4.

Tables 2, 3, and 4 present the characteristics of the state variables: the initial settings, differences among agents, and how values change with a change in time step. The initial settings are the values assigned to the state variables of the objects when the objects are created. The difference among agents shows whether the values are the same or different among the agents. The change in time step indicates whether the values are time dependent.

Table.2 State variables for agents' behavioral category

		Characteristics of state variable			
Category of entity	**State Variable**	**Initial setting**	**The difference among agents**	**Change in time step**	**Description**
Agents in general	***Agent id***	Sequential	Different	Invariable	Identification number of agents
	Agent type id	Specified	Same / Different	Invariable	Specified number of agent type for consumers, retailers, raw material makers, an equipment maker, a bank and a governemnt
	Cash	At random	Different	Variable	Cash possessed by agent, which is variable each time step.
	Deposit	At random	Different	Variable	Deposit possessed by agent in the bank.
Buyers	***Weight of utility for each product class***	At random	Different	Invariable	The weight of utility for each product class of goods to purchase. Initially assigned at random between 0 and 1 for each agent. Non-zero value is assigned for all of the product classes in case of government and for 2 classes in case of other agent types of buyers.
	The number of goods purchased	--	Different	Variable	The number of goods purchased for each product class, which is assumed to be reset at the beginning of each time step.
	Exponent in utility function	At random	Different	Invariable	The exponent of the number of goods to purchase in utility function.
Enterprises	***Number of employee***	At random or specified	Different	Variable or Invariable	The number of agents working in an enterprise including workers and an executive.
	Agent id of an executive	At random	Different	Invariable	The agent id of the consumer specified as an executive.
	Agent id of workers	At random	Different	Variable	The agent id of the consumer specified as a worker.
	Fixed wages	At random	Different	Invariable	The amount of fixed wages to be paid to each employee including an executive
	Bonus ratio	Specified	Same	Invariable	The ratio of bonus to be paid for workers with respect to before-tax profit.
	Executive compensation ratio	Specified	Same	Invariable	The ratio of executive compensation paid for the executive with respect to after-tax profit. In this study, executive compensation is assumed to be an extra bonus paid for an executive, which is defined as the executive compensation ratio multiplied by of after-tax profit of an enterprise.
Producers	***Product class id***	At random	Different	Variable	The class id of product to produce.
	A list of goods in the market	--	Different	Variable	A list of the objects of the producer's supplied goods in the market. At the beginning of each period, it shows a list of unsold stocks in the market. During each period, it increases with an increment number of produced goods and decreases with an increment number of goods sold during the period.
	Dismissal flag	--	Different	Variable	The flag number for decision making of the dismissal of a worker. When the profit is negative or positive at a certain period, the dismissal flag is increased or decreased by 1. When the dismissal flag reaches a critical flag number for dismissal, the producer fires one employee who is selected at random.
	Quit-production flag	--	Different	Variable	The flag value for decision making regarding stoppage of production. When the produced products with a certain class all remain unsold, quit-production flag is increased by 1. when it is not, it is decreased by 1. When it reaches a critical flag number to quit production, the producer stops its production.

Table.3 State variables peculiar to each type of agent

		Characteristics of state variable			
Entity	**State Variable**	**Initial setting**	**The difference among agents**	**Change in time step**	**Description**
Consumer	***Working place***	At random	Different	Variable	The agent id of the enterprise or government the consumer works for.
	Marginal propesity to consume	Specified	Same	Invariable	The proportionality constant of disposable income after tax for the budget for purchasing consumption goods.
	Basic consumption	Specified	Same	Invariable	The minimum budget for purchasing consumption goods when withdrawal of deposit is assumed to be 0.
	Withdrawal ratio	At random	Different	Variable	The ratio of money withdrawn from the deposit to purchase consumption goods. It is randomly assigned for each agent at every period during the simulation.
Retailer and Raw material makers	***Prurchasing ratio***	Specified	Same	Invariable	Percentage of accumulated profit for buying consumption goods. The budget for consumption is determined as the purchasing ratio multiplied by accumulated profit.
	Proportionality constant of production function	At random	Different	Invariable	The proportionality of Cobb Douglas's production function. This proportionality is express the technical capabilities of each enterprise in present model.
	Investment flag	--	Different	Variable	The Flag number for deciding investment. It increases or decreases by 1 depending on the producer's own stock in the market.
	Upper limit of the number of loans	Specified	Same	Invariable	The upper limit for the number of issuance of long term loans at a time, the funds of which are required for investment and financed by the bank. They cannot invest in equipment when their number of loans has already reached this value.
Equipment maker	***Price of equipment***	Specified	--	Invariable	The Price of equipment of one unit of equipment.
	Limit number of production per each period	Specified	--	Invariable	The upper limit for production per period. When it receives orders more than this value, it does not meet the demand by rejecting the order.
Bank	***Repayment period***	Specified	--	Invariable	Repayment period of long term loan.
	Interst rate on loans	Specified	--	Invariable	Interest rate on loans of producer for investment
	Interest rate on deposits	Specified	--	Invariable	Interest rate on deposits of producer and consumer.
Government	***Income tax rate***	Specified	--	Invariable	Income tax rate levied on consumers' income
	Corporation tax rate	Specified	--	Invariable	Corporation tax rate levied on producer's profits
	Salary for public workers	--	--	Variable	Salary for workers who work for government which is determined as the average of payment for private-sector employees including bonus
	Ratio of market purchasing	Specified	--	Invariable	Ratio of the budget for purchasing consumption goods in the market.
	Ratio of firm subsidy	Specified	--	Invariable	Ratio of the budget for subsidizing firms with no limitation of its use.

Table.4 State variables of other types of entities

		Characteristics of state variable		
Entity	**State Variable**	**Initial setting at create**	**Change in time step**	**Description**
Consumption goods	***Product class number***	Specified by the producer	Invariable	Class number of product determined by the producer
	Price	Specified by the producer	Variable	It is determined by the producer at every period.
	Seller's number	Specified by the producer	Invariable	The agent number of the producer who produced it
	Buyer's number	No setting	Invariable	The agent number of the agent who bought it. It is determined when it was bought.
Material goods	***Product class number***	Specified by the producer	Invariable	Class number of product determined by the producer
	Price	Specified by the producer	Variable	It is determined by the producer at every period.
	Seller's number	Specified by the producer	Invariable	The agent number of the producer who produced it
	Buyer's number	--	Invariable	The agent number of agent who bought it. It is determined when it was bought.
Market	***A list of consumption goods***	--	Variable	A list of consumption goods objects it holds
	A list of material goods	--	Variable	A list of material goods objects it holds

3. Process overview and scheduling

The present model consists of three submodels: a funds circulation submodel, a price equilibrium submodel, and an investment submodel. The funds circulation submodel constitutes the fundamental structure of the model in which the latter two submodels are implemented. The model consists of three processes: initialization, where the objects of the entities are created and initialized; the sequence of seven actions performed by agents during each time step; and the calculation of the average GDP and other statistical data of macroeconomic indicators. The seven steps comprise the actions at the beginning of every time step, the production of raw materials, the production of consumption goods, purchasing of consumption goods, payment of wages, actions for investment, and the actions at the end of every time step. The pseudocode that describes this process is given in Fig 2, and the sequential events conducted by each type of entity during each time step are described in Fig. 3.

for period=0 to period bound
  if period=0 then
    Initialize agents, set parameters, set initial conditions.
    Agents do actions in the similar way as period>0 with some exceptions.
  else
    1. Agents pay unpaid tax for the previous period, make a budget plan for expenditures.
    2. Raw material makers decide the amount and price of products.
       They produce raw materials of several types, supply them to the market.
    3. Retailers decide the amount and price of products, purchase raw materials.
       They produce products of several types, supply them to the market.
    4. Agents except for the equipment maker and the bank purchase products in the market.
    5. Retailers and raw material makers judge the necessity of investment on the basis of
       total sales in the previous periods. If necessary, they invest in equipment.
    6. Each firm pays wages and executive compensation for workers and the executive.
       Government pays wages for public workers.
    7. Each agent settles its accounts, calculating income or profit for the current term,
       based on which the amount of tax to be paid is determined.
    If necessary, each retailer dismisses a worker on the basis of profits for previous periods
       or decides to stop production of a certain type of products on the basis of its total sales.
  Calculate GDP and input-output table by summing the accounts data of all agents.
end for
Calculate average GDP for all periods.

Fig.2 Pseudocode of the model

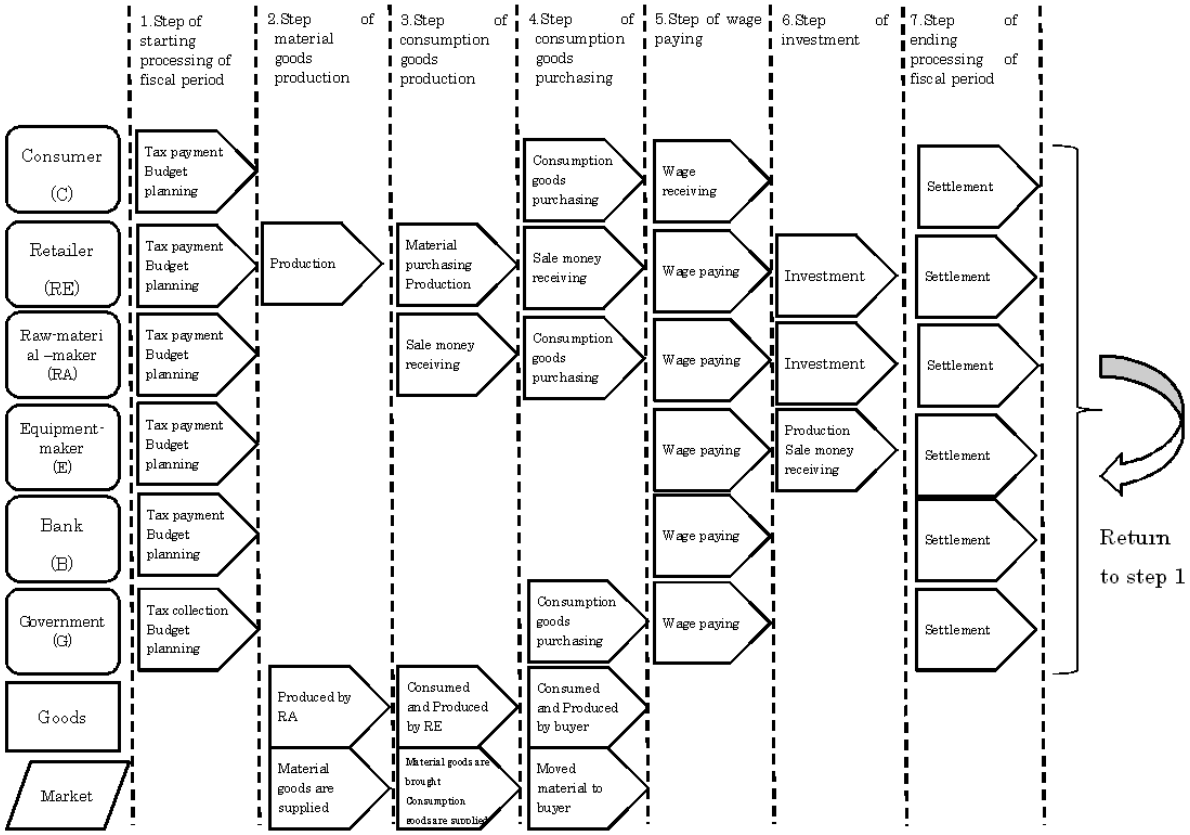

Fig.3 The sequential events in each time step for each type of entity

4. Design concepts

***Basic principles*:**

The general concept underlying the model design of ABM is that the behavior of an artificial economic system can mimic the real-world behavior if the model structure and the structure of the real systems have a homomorphic relationship. This relationship is considered to be fulfilled when the structural factors of the modeled system are essentially the same as those of the real system with respect to the relevant macroscopic economic phenomenon. Therefore, ABM can be useful in describing the mechanism of a macroeconomic phenomenon by performing controlled experiments in which only one factor of interest varies at a time, while holding other factors constant. In this way, ABM clarifies the structural conditions necessary for the model to reproduce the macroeconomic phenomenon being studied.

Emergence:
The modeled artificial systems should include heterogeneous and autonomous agents. Their behavioral rules might be similar, but the values of their state variables should be different. Therefore, the heterogeneous agents behave differently and interact with each other. Macroscopic phenomena emerge from these actions and interactions, which affects the microscopic behavior of the agents, resulting in a micro-macro link in the dynamics of the systems. In this way, artificial economic systems can behave as complex systems.

Adaptation:
Retailers and raw material makers adjust the price and number of products they supply to the market by gauging the demand in the market. To do so, they observe the number of stock items that remain unsold at the end of each time step. These producers also use the market demand to adjust their production capacity and number of employees. In this way, the artificial economic systems in this study possess an internal adaptation mechanism.

Objectives:
Consumers and producers hold their own objective functions, such as maximizing utility or profits.

Prediction:
Retailers and raw material makers predict the total sales of their goods based on the sales figures from the ten most recent periods. Based on this prediction, they decide on the amount of production in the next time step so that the probability of goods being out of stock is less than 5%. When they decide to invest in equipment, they first predict the financial benefit of the investment by estimating the increase in profit gained from a one-unit increase of equipment and the subsequent increase in production capacity.

Sensing.
Retailers and raw material makers gauge the market demand by observing the amount of goods still in stock at the end of each time step. They also calculate the optimal number of employees based on the profit of the current term, as well as the potential financial benefit of increasing or decreasing the number of employees and, therefore, their production capacity.

Interaction.
The price equilibrium is loosely attained by the interaction between agents' purchasing actions and producers' actions when adjusting their production levels and product prices. In addition, the circulation of funds and the emergence of various macroeconomic indicators,

such as GDP, are the result of the actions of agents and their interactions. The investment behavior is also a result of the interaction between buyers and producers.

Stochasticity:
Various state variables are randomly defined at the start of the simulation or during the simulation. Typical examples are agents' initial funds, state variables that distinguish agents (e.g., product classes), production capacities, utility weights, and a consumer's workplace. These are defined using random numbers with a uniform distribution.
The order of agents' actions for the same type of agent is also defined by shuffling the set of agent id numbers using a uniform random number at every time step..

Observation:
At the end of each time step, each agent settles its account using the double-entry bookkeeping method. An input-output table for the artificial system is defined by summing the calculated data for all agents. The macroeconomic indicators, such as GDP, tax revenue, total funds for each type of agent, total salaries paid by producers, and the total number of investments, are calculated based on the input-output table and other account data of the agents. In addition, statistical data, such as the total number of goods produced or bought during each time step, the average price of products, and the amount of funds circulated between the bank and other types of agents, are also calculated at the end of each time step. The average values of these data for the overall simulation can also be obtained and used for various types of analysis.

5. Initialization

All state variables of the agents are initialized when the agent objects are created. These initial values are described in Table 5 and Table 6. An Agent id is sequentially assigned for each agent, but this number is only used to distinguish agents. Each agent object is initially assigned randomly to one of the types of agents.

Table.5 Initialization of state variables for agent's behavioral category

Category of entity	State Variable	Description	Initialization
Agents in general	Agent id		--
	Agent type id		--
	Cash	--Consumer	30000-50000
		--Retailer and raw material maker	80000-160000
		--Equipment maker	200000-220000
		--Bank	96000000-104000000
		--Government	10000
	Deposit		0
Buyers	Weight of utility for each	--Consumer:	0-1
	product class	--Producer and government	0
	Exponent in utility function		0-1
Enterprises	Number of employee	--Retailer	4-5 at random
		--Raw material maker	3
		--Equipment maker	2
		--Bank	1
	Fixed wages		7000-7500
	Bonus ratio		75%
	Executive compensation ratio		95%
Producers	Product class id		2 classes are randomly assigned for each agent between 1 and 6 at the start of simulation
	Dismissal flag	--Dismissal flag	0
		--Critical flag number for dismissal	5
	Quit-production flag	--Quit-production flag	0
		--Critical flag number to quit production	20

Table.6 Initialization of state variables peculiar to each type of agents

Entity	State Variable	Description	Initialization
Consumer	Working place		--
	Marginal propesity to consume		70%
	Basic consumption		3000
	Withdrawal ratio		0-50%
Retailer and raw material makers	Prurchasing ratio		70%
	Proportionality constant of production function	--Retailer	8-18
		--Wholesaler	50-150
	Investment flag	--Investment flag	0
		--Critical number of investment flag	20
	Upper limit of the number of loans		3
Equipment maker	Price of equipment		500000
	Limit number of production per each period		4
Bank	Repayment period		120
	Interst rate on loans		3%
	Interest rate on deposits		0.5%
Government	Income tax rate		20%(Standard)
	Corporation tax rate		20%(Standard)
	Salary for public workers		--
	Ratio of market purchasing		60%(Standard)
	Ratio of firm subsidy		40%(Standard)

6. Input data

No data from the real system is used as input data for the simulation.

7. Submodels

7.1 Funds circulation submodel

This submodel constitutes the fundamental structure of the model, the outline of which is described in the pseudocode presented in Fig. 2. The basic principles of the circulation of funds and additional behavioral rules are presented below.

(1) Basic principles of funds circulation.

Consumers work for one of the other agents, receive wages, buy consumption goods produced by retailers, and pay income tax to the government. Retailers produce consumption goods using raw material goods supplied by raw material makers, where minimum units of supply chain processes are implemented in the model. The behavioral rules for the strategies of consumer purchasing and producers' production are described in the price-equilibrium submodel. Retailers and raw material makers invest in equipment when doing so will increase their profit. The investment strategies are described in investment submodel.

The government levies income tax and corporation tax, pays wages to public employees and conducts public expenditure, comprising market purchasing as an extreme case of efficient public spending and firm subsidies as an extreme case of inefficient public spending.

In this way, funds circulate among agents in the artificial economic systems as a result of agents' actions and interactions.

(2) Related behavioral rules.

(2)-1 Agents' behavioral rules for determining their consumption budget.

Every agent, other than the bank, determines a consumption budget at the beginning of each time step. The definitions of the budget are different each type of agent.

For the consumer agent:

$$E_b^t = a + bI^t + r_{wd}^t D^t$$

where, $\boldsymbol{E_b^t}$: Consumer's consumption budget

$\boldsymbol{a}$: Basic consumption; $\boldsymbol{b}$: Marginal propensity to consume;

$\boldsymbol{I^t}$: after-tax income; $\boldsymbol{r_{wd}}$: Withdrawal ratio; $\boldsymbol{D^t}$: Bank deposit

For the producer agent: Purchasing ratio multiplied by the amount of internal funds

For the government agent:

$$E_b^t = E_{all\_b}^t - wage_G^t$$

where, E_b^t: Total public expenditure budget

$E_{all\_b}^t$: Total amount of tax revenue

$wage_G^t$: Total salaries paid to public employees

The budgets for market purchasing and for firm subsidies are defined as the ratio of the respective amount of public expenditure to the total budget.

(2)-2 Payment of salaries.

○ Salaries paid by enterprises.

Each enterprise agent pays a fixed salary, a bonus, and executive compensation. The total amount paid as salaries depends on both the before-tax profit and accumulated profit, as given below:

$$E_w^t = \begin{cases} W_f & if \quad \pi^{t-1} < 0 \\ W_f + W_b^{t-1} & if \quad \pi^{t-1} > 0 \ and \ AC < 0 \\ W_f + W_b^{t-1} + EC^{t-1} & if \quad \pi^{t-1} > 0 \ and \ AC > 0 \end{cases}$$

where, $\boldsymbol{E_w^t}$: Total salary amount

W_f: Fixed salary

$\boldsymbol{W_b}$: Bonus

$\boldsymbol{EC}$: Executive compensation

$\boldsymbol{AC}$: Accumulated profits

The total amount of salaries paid to workers or to executives is given below.

$$W_C^t = W_f + W_b^{t-1} / ne \quad for \text{ workers}$$

$$W_C^t = W_f + W_b^{t-1} / ne + EC^{t-1} \quad for\ executives$$

where, $\boldsymbol{W_c^t}$: Total salaries paid to workers or to executives

ne: The number of employees

○ Salaries paid by the government

The government pays fixed salaries to public workers based on the previously determined budget for wages.

(2)-3 Agents' behavioral rules for settling accounts at the end of each time step

○ The rules for consumers

Consumers define the amount of income tax to be paid based on their income and remember this as the amount of unpaid tax:

$$Tax_i = W_C^t \, r_{i\_tax}$$

where, $\boldsymbol{Tax_i}$: The amount of income tax

$\boldsymbol{r_{i\_tax}}$: The income tax rate

A part of consumers' income, including unpaid tax, is kept on hand as cash and deposited in the bank, as given below:

$$deposit = (1-b)(W_C^t(1-r_{i\_tax})) - a$$

○ The rules for producers

Producers define their profit based on total sales and total expenses:

$$Pr_p^t = S^t - (W_f + \sum co^t + int^t + dep^t)$$

where, $\boldsymbol{Pr_p}$: The profit before bonus

$\boldsymbol{S}$: Total sales

$\boldsymbol{\Sigma co}$: Total expenses for raw materials

int: Interest to be paid

dep: Depreciation expenses

Producers define the amount to be paid as bonuses to employees based on the profit before bonuses, as given below. They remember this as the amount of unpaid bonuses:

$$W_b^t = Pr_p^t r_{bonus}$$

where, $\boldsymbol{r_{bonus}}$: The ratio of bonus

Based on this value, they define their before-tax profit as given below:

$$Pr_{a\_tax}^t = Pr_p^t(1 - r_{bonus})$$

where, $\boldsymbol{Pr^t_{a\_tax}}$: The before-tax profit

Then, they calculate the amount of corporation tax to be paid and remember this as the amount of unpaid tax:

$$Tax_c = Pr_p^t(1 - r_{bonus})r_{c\_tax}$$

where, Tax_c : The amount of corporation tax

$\boldsymbol{r_{c\_tax}}$: The rate of corporation tax

Based on this value, they define their after-tax profit and executive compensation, and remember this as unpaid executive compensation:

$$EC^t = Pr_p^t(1 - r_{bonus})(1 - r_{c\_tax})r_{exec}$$

where, $\boldsymbol{r_{ecex}}$: The ratio of executive compensation

Extracting the executive compensation from their after-tax profit enables producers to define their accumulated profit, as given below:

$$Ac^t = Ac^{t-1} + \mathrm{Pr}_p^t(1 - r_{bonus})(1 - r_{c\_tax})(1 - r_{ecex})$$

○ The rules for the government

The government defines the total amount of tax revenue and expenses, and passes the resultant money on to the next period.

(2)-4 Others

○ The rules for dismissal

At the end of each time step, the retailer fires one of its employees if its dismissal flag reaches a critical value. The employee to be fired is selected at random and is assigned to the producer with the largest accumulated profit.

○ The rules for stopping production and for bankruptcy

At the end of each time step, the producer stops production of a certain class of product if its flag reaches a critical value. When a producer stops all its product classes, it then goes bankrupt and a new producer object is created with new initial variables.

7.2. Price equilibrium submodel

The present model mimics the price equilibrium in the market according to the following two principles.

(1) Lowest-price-oriented purchasing strategy by buyers.

All buyers purchase consumption goods within the limits of their consumption budget. If there are products within same product class, but with different prices, they will select the cheapest of them. The consumption goods bought are indexed by buyer's id and are removed from the market and moved to the buyer.
In addition, consumers purchase products to maximize their utility within the limit of their consumption budget.

$$\max u = \sum_i w_i x_i^{\alpha} \qquad s.t. \sum_i p_i^t x_i \le E_b^t$$

where, w_i : The weight of utility for each product of class i
x_i: The number of products to purchase
p_i: The price of a product
α: An exponent of x_i

(2) Stock-control-oriented production strategy by sellers.
(2)-1 The behavioral rules used by producers to determine the price of their products.
The price of a product is defined according to the number of products in stock and the amount bought in the market.

$$p^t{}_i = \begin{cases} (1+\gamma_i)\, p^{t-1}{}_i & if \quad s_i^{t-1} = 0 \\ (1-\gamma_d)\, p^{t-1}{}_i & if \quad s_i^{t-1} > 0 \ and \ \ p^{t-1}{}_i < p_{avei}^{t-1} \end{cases}$$

where, γ_i: The ratio of a price increasing
γ_d: The ratio of a price decreasing
$s^{t-1}{}_i$: The amount of goods in stock at the end of previous period
$p^{t-1}{}_{avei}$: The average price of goods bought in the market in the previous period

(2)-2 The strategy for amount to be produced (the production plan)
--The number of products to be produced in a given period is defined so that the probability of goods being out of stock is 5%:

$$q_{\mathrm{si}}^t = q_{\mu i}^t + 1.65 q_{\sigma i}^t$$

where, q^t_{si} : Target number of goods in stock
$q^t_{\mu i}$: Average sales during the past ten periods
$q^t_{\sigma i}$: Sigma of total sales during the past ten periods

--Producers decide on the number of products to produce according to the number of products in stock, adjusting their target as shown below.

$$q^t{}_i = \begin{cases} q_{si}^t (1+\varepsilon) & if \quad s_i^{t-1} = 0 \\ q_{si}^t (1-\varepsilon) - s_i^{t-1} & if \quad s_i^{t-1} > 0 \end{cases}$$

$$If \ \ q^t{}_i > Y_i(K,L) \ \ q^t{}_i = Y_i(K,L)$$

where, q^t_i: The amount of production
ε: The ratio of changing amount of production
$Y_i(K, L) = A_i K^{\alpha} L^{1-\alpha}$: Production capacity
K: The number of units of equipment for production
L: The number of employees
A: A proportionality constant

7.3. Investment submodel

(1) Producers' behavioral rules for investment decisions

The retailer or raw material maker decides to invest when the three conditions listed below are fulfilled. Once the agent decides to invest, it becomes a candidate for investment and is included in the list of candidates owned by an equipment maker.

Conditions for investment:

1) The investment flag number exceeds a critical value for investment.

2) The financial benefit from an increase in one unit of equipment is positive, as given below.

$$\Delta\pi_K = \max_i[(p_i^t - c_i^t)\{Y_i(K+1,L) - Y_i(K,L)\} - (r_0 + 1/N)F] > 0$$

where, $\boldsymbol{p_i}$: The price of goods of product of class $\boldsymbol{i}$

$\boldsymbol{c}$: The variable cost per unit product

$\boldsymbol{r_0}$: The borrowing interest rate

$\boldsymbol{F}$: The borrowed money required to buy one unit of equipment

$\boldsymbol{N}$: The repayment period.

3) The accumulated profit at the end of current term is greater than half the necessary funds for investment.

(2) The behavioral rules for equipment makers.

Equipment makers randomly select one of the candidates for investment and sell that agent a unit of equipment. If there is more than one candidate, the equipment maker continues until the number of units reaches the equipment maker's production capacity.

(3) Producers' behavioral rules for financing and buying equipment.

The selected retailer or raw material maker purchases one unit of equipment. Before purchasing, the agent finances half the necessary funds using internal funds from accumulated profits and the other half from the bank. After purchasing, the retailer or raw material maker renews its production capacity by increasing the number of units of equipment by one in the Cobb-Douglas-type equation.